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Solving Time Fractional Sharma−Tasso−Olever Equation using Fractional Complex Transform with Iterative Method

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors made significant contribution. Both authors read and approved the final manuscript.

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Abstract

In this paper, we consider time fractional Sharma−Tasso−Olever equation based on Jumarie's modified Riemann-Liouville derivatives. By using fractional complex transform the time fractional Sharma−Tasso−Olever equation reduces to nonlinear partial differential equation and then new iterative method is applied to get approximate solutions. The Numerical results and the graphs of the solutions are compared with the exact solutions to verify the applicability, efficiency and accuracy of the method.

Keywords: Fractional complex transforms; modified Riemann−Liouville definition; new iterative method; time fractional Sharma− Tasso− Olever equation.

2010 Mathematics Subject Classifications : 26A33, 33E12, 34A08, 35R11.

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1 Introduction

The theory of fractional calculus is as old as classical itself. In recent years many researchers have been attracted towards fractional differential equations due to its extensive applications in wide areas of science and engineering such as physics, chemistry, mechanics, electrical networks, astronomy, diffusion, viscoelastic fluid, signal and image processing, reaction processes etc. [1, 2, 3, 4]. Various linear or nonlinear fractional partial differential equations has been successfully used to model a variety of dynamical systems, mechanical systems, physical or biological processes, anomalous diffusive and sub diffusive systems, unification of diffusion and wave propagation phenomena. These problems are modelled using different types of fractional derivative operators which includes Riemann-Liouville definition, Caputo definition, Riesz definition, Riesz-Feller definition, and the Jumaries modified Riemann-Liouville definition which is recently proposed by Jumarie [5, 6, 7]. Solving fractional differential equations is turn out to be extensive area of research and interest for researchers from various fields. The commonly used analytical and numerical methods to solve these equations are the Adomian decomposition method ADM[8, 9], Variational iteration method VIM [10, 11], Homotopy-perturbation method HPM [12], Homotopy analysis method [13], Finite element method [14], Finite-difference method [15, 16], RBF-based collocation methods [17], Operational matrix method [18], Boundary particle method (BPM) [19], etc. Recently, one of the most reliable and effective technique to solve linear and nonlinear functional equations is proposed by Daftardar-Gejji and Jafari [20, 21, 22, 23] known as new iterative method (NIM).

In literature several effective transforms such as the laplace transform, the travelling wave transform, the fourier transform, the backlund transformation, the integral transform, and the local fractional integral transforms proposed to solve numerous problems. In recent times, the fractional complex transform has been proposed in [24, 25, 26] to convert fractional-order differential equations based on modified Riemann-Liouville derivatives [6] into integer order differential equations, and then reduced equations can be solved by simple calulus.

In recent times, both mathematicians and physicists have devoted considerable effort to the study of explicit solutions to nonlinear fractional partial differential equations. Specially the theory of solitons which is studied in various forms such as analysing topological solitons known as shock-waves, singular solitons that are also known as rogue waves in oceanography and optical rogons in nonlinear optics. The Sharma-Tasso-Olver equation is one such type of nonlinear equation that gives shock-wave solutions or topological soliton solutions. Many methods of integration are successfully implemented to extract solitary wave solutions and other solutions to the classical and fractional Sharma-Tasso-Olver equation.[27, 28, 29, 30, 31, 32, 33, 34]

In the present paper, we have applied fractional complex transform using new iterative method to obtain approximate solutions of time fractional Sharma−Tasso−Olever equation as given below :

$$\frac{\partial^\alpha u}{\partial t^\alpha} + 3\rho\left(\left[\frac{\partial u}{\partial x}\right]^2 + u^2\frac{\partial u}{\partial x} + u\frac{\partial^2 u}{\partial x^2}\right) + \rho\frac{\partial^3 u}{\partial x^3} = 0 \qquad t>0, \quad 0<\alpha\le 1 \tag{1.1}$$

where ρ is an arbitrary real constant and u(x,t) is the unknown function dependent on the variables x and t. Eq.(1.1) reduces to the classical nonlinear Sharma−Tasso −Olever equation for $\alpha = 1$

The rest of this paper is organized as follows. In Sections 2, basic definitions are presented. In Sections 3 and 4 we give an analysis of the fractional complex transform and new iterative method. The numerical results and graphs for the time fractional Sharma−Tasso−Olever equation are presented in Section 5. Finally, we give our conclusions in Section 6.

2 Basic Definitions

In literature there are many definitions on fractional derivatives [1−4] but the most frequently used are as below

2.1. Riemann−Liouville definition:

$$D_x^\alpha(f(x)) = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dx^m}\int_0^x (x-t)^{m-\alpha-1}f(t)dt \tag{2.1}$$

where $m-1 \le \alpha < m, \quad m \in N \cup \{0\}$

2.2. Caputo's definition:

$$D_x^\alpha(f(x)) = \frac{1}{\Gamma(m-\alpha)}\int_0^x (x-t)^{m-\alpha-1}\frac{d^m}{dx^m}f(t)dt \tag{2.2}$$

where $m-1 \le \alpha < m, \quad m \in N \cup \{0\}$

2.3. Jumaries definition:[5]

$$D_x^\alpha(f(x)) = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dx^m}\int_0^x (x-t)^{m-\alpha-1}[f(t)-f(0)]dt \tag{2.3}$$

2.4. He's fractional derivative:[25]

$$\frac{\partial^\alpha u}{\partial x^\alpha} = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dx^m}\int_{x_0}^x (t-x)^{m-\alpha-1}[u_0(t)-u(t)]dt \tag{2.4}$$

where $u_0(x,t)$ is the solution of its continuous partner of the problem with the same initial condition of the fractal partner.

3 Fractional Complex Transform

Fractional complex transform was proposed by Li and He [24] is a very simple solution procedure to convert the fractional differential equations into ordinary differential equations, hence all the analytical methods devoted to the advanced calculus can be easily applied to the fractional calculus.

Consider the following general fractional differential equation

$$f(u, u_t^{(\alpha)}, u_x^{(\beta)}, u_y^{(\gamma)}, u_z^{(\lambda)}, u_t^{(2\alpha)}, u_x^{(2\beta)}, u_y^{(2\gamma)}, u_z^{(2\lambda)}, \cdots) = 0 \tag{3.1}$$

where $u_t^{(\alpha)} = \frac{\partial^\alpha u(x,y,z,t)}{\partial t^\alpha}$ denotes modified Riemann−Liouville derivatives.

$0 < \alpha \le 1, \; 0 < \beta \le 1, \; 0 < \gamma \le 1, \; 0 < \lambda \le 1$

Introducing the following fractional complex transforms

$$T = \frac{qt^\alpha}{\Gamma(1+\alpha)},$$

$$X = \frac{px^\beta}{\Gamma(1+\beta)},$$

$$Y = \frac{ly^\gamma}{\Gamma(1+\gamma)},$$

$$Z = \frac{kz^\lambda}{\Gamma(1+\lambda)},$$

where p, q, k and l are unknown constants.Using the basic properties of the fractional derivative and the above transforms, we can convert fractional derivatives into partial derivatives as:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = q\frac{\partial u}{\partial T}$$

$$\frac{\partial^\beta u}{\partial x^\beta} = p\frac{\partial u}{\partial X}$$

$$\frac{\partial^\gamma u}{\partial y^\gamma} = l\frac{\partial u}{\partial Y}$$

$$\frac{\partial^\lambda u}{\partial z^\lambda} = k\frac{\partial u}{\partial Z}$$

Therefore, the fractional differential equations are easily converted into partial differential equations, so that anyone acquainted with simple calculus can deal with fractional calculus without any difficulty which can be solved further by new iterative method.

4 Analysis of New Iterative Method

Daftardar-Gejji and Jafari [20] have introduced a new iterative method (NIM) which is simple to understand and easy to implement in solving nonlinear equations.
In this method, consider a functional equation of the form

$$u = f + L(u) + N(u) \tag{4.1}$$

Where f is a given function, L and N are given linear and non-linear operator. Let u be a solution of Eq.(4.1) having the series form:

$$u = \sum_{i=0}^{\infty} u_i \tag{4.2}$$

Since L is linear $L(\sum_{i=0}^{\infty} u_i) = \sum_{i=0}^{\infty} L(u_i)$. The nonlinear operator here is decomposed as :

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\} \tag{4.3}$$

$$= \sum_{i=0}^{\infty} G_i \tag{4.4}$$

where $G_0 = N(u_0)$ and $G_i = \left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}$, $i \geq 1$

hence Eq.(4.1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} G_i \tag{4.5}$$

Further consider the recurrence relation as:

$$u_0 = f$$

$$u_1 = L(u_0) + G_0$$

$$u_{m+1} = L(u_m) + G_m, \quad m = 1, 2, \cdots \tag{4.6}$$

Then

$$(u_1 + u_2 + \cdots + u_{m+1}) = L(u_0 + u_1 + \cdots + u_m) + N(u_0 + u_1 + \cdots + u_m), \qquad m = 1, 2, \cdots$$

and $u = f + \sum_{i=1}^{\infty} u_i$

The k−term approximate solution is given by $u = u_0 + u_1 + u_2 + \cdots + u_{k-1}$

For the details of the condition for convergence of the series $\sum u_i$ we refer to the reader to [35].

5 Numerical Application

In this section, to illustrate the efficiency and accuracy of the combination of fractional complex transform and new itertive method, we have obtained approximate solution of time fractional Sharma−Tasso−Olever equation based on Jumarie's modified Riemann−Liouville derivatives. All computations are performed with the help of Mathematica.

Consider Eq. (1.1) with initial condition

$$u(x,0) = \frac{2k(w + tanh(kx))}{1 + w\ tanh(kx)}, \qquad k, w \in C \tag{5.1}$$

where the exact solution of (1.1) is given in [30] as

$$u(x,t) = \frac{2k\left(w + tanh(k(x - 4ak^2t))\right)}{1 + w\ tanh(k(x - 4ak^2t))} \tag{5.2}$$

we consider the transformation

$$T = \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{5.3}$$

hence

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = \frac{\partial u}{\partial T}$$

Using this transformation (5.3) in Eq.(1.1) we get

$$\frac{\partial u}{\partial T} + 3\rho\left(\left[\frac{\partial u}{\partial x}\right]^2 + u^2\frac{\partial u}{\partial x} + u\frac{\partial^2 u}{\partial x^2}\right) + \rho\frac{\partial^3 u}{\partial x^3} = 0 \tag{5.4}$$

Applying Integral operator I_T on both side of Eq.(5.4) and using initial condition we obtain the relation

$$u(x,T) = u_0(x,T) + L(u) + N(u)$$

where

$$L(u) = I_T\left(-\rho\frac{\partial^3 u}{\partial x^3}\right) \quad \text{and} \quad N(u) = I_T\left(-3\rho\left(\left[\frac{\partial u}{\partial x}\right]^2 + u^2\frac{\partial u}{\partial x} + u\frac{\partial^2 u}{\partial x^2}\right)\right)$$

Taking series solution as $u(x,T) = \sum_{i=0}^{\infty} u_i(x,T)$ and using iteration formula (4.3) and (4.6) with initial condition (5.1) we get

$$u_0 = \frac{2k(w + tanh(kx))}{1 + w\ tanh(kx)} \tag{5.5}$$

$$u_1 = \frac{8\rho k^4(-1 + w^2)\ T}{(cosh(kx) + w\ sinh(kx))^2} \tag{5.6}$$

$$u_2 = \frac{\left(4\rho^2 k^7(-1+w^2)(w\, cosh(kx)+sinh(kx))\ T^2\right)\left(3+192\rho^2k^6T^2-6w^2-384\rho^2k^6w^2\ T^2\right)}{(cosh(kx)+wsinh(kx))^7}$$
$$+\frac{\left(-4(-1+w^4)cosh(2kx)+8wsinh(2kx)\right)\left(4\rho^2k^7(-1+w^2)(w\, cosh(kx)+sinh(kx))\ T^2\right)}{(cosh(kx)+wsinh(kx))^7}$$
$$+\frac{\left((1+6w^2+w^4)cosh(4kx)-8w^3sinh(2kx)\right)\left(4\rho^2k^7(-1+w^2)(w\, cosh(kx)+sinh(kx))\ T^2\right)}{(cosh(kx)+wsinh(kx))^7}$$
$$+\frac{\left(4wsinh(4kx)+4w^3sinh(4kx)\right)\left(4\rho^2k^7(-1+w^2)(w\, cosh(kx)+sinh(kx))\ T^2\right)}{(cosh(kx)+wsinh(kx))^7} \quad (5.7)$$

In the same manner the remaining components of the Eq. (4.6) can be obtained from Mathematica software. Substituting Eq.(5.3) in the Eq.(5.6) and (5.7) we get three term approximate solution of Eq.(1.1) as

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t)$$

Fig. 1 shows the surfaces of approximate solution obtained from new iterative method of Eq.(1.1) when $\rho = k = \alpha = 1$ and $w = \frac{1}{2}$ and Figure 2 shows the surfaces of approximate solution of Eq.(1.1) $\rho = k = 1$, $\alpha = 0.5$ and $w = \frac{1}{2}$

Fig. 3 shows the surfaces of approximate solution obtained from new iterative method of Eq.(1.1) when $\rho = \alpha = 1$, $k = 0.1$ and $w = \frac{1}{2}$ and Fig. 4 shows the surfaces of approximate solution of Eq.(1.1) for $\rho = 1$, $\alpha = 0.5$ and $k = 0.1$ and $w = \frac{1}{2}$

In Fig. 5 , we study the plots obtained from the 3rd−order approximate solution for $\alpha = 1,\ 0.9,\ 0.8,\ 0.7$, $\rho = k = 1$ and $w = \frac{1}{2}$, and in comparison with exact solution Eq. (5.2) at $x = 2$. We can see that the approximate solution for $\alpha = 1$ has the similar shape as the exact solution for $0 \leq t \leq 0.1$,

In Fig. 6 , we study the plots obtained from the 3rd−order approximate solution for $\alpha = 1,\ 0.9,\ 0.8$, $\rho = k = 1$ and $w = \frac{1}{2}$, and in comparison with exact solution Eq. (5.2) at $t = 0.1$ for $-5 \leq x \leq 5$. We can see that the shape of curve of approximate solution for $\alpha = 1$ coincides with shape of the exact solution for $0 \leq t \leq 0.1$. Therefore, based on these present results, we can say that new iterative method is more accurate and efficient.

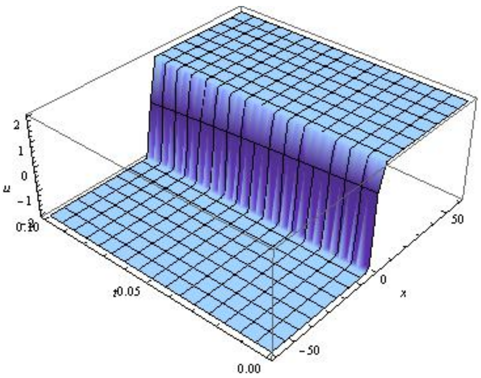

Fig. 1. Approx. soln. $\alpha = 1,\ k = 1$

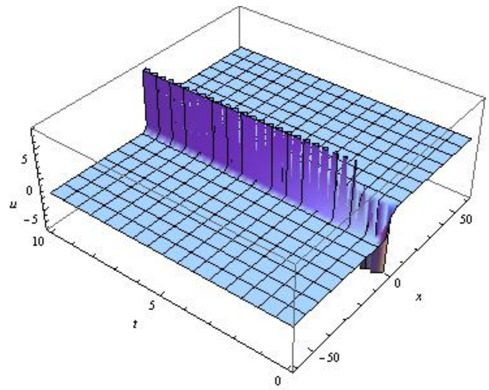

Fig. 2. Approx. soln. $\alpha = 0.5,\ k = 1$

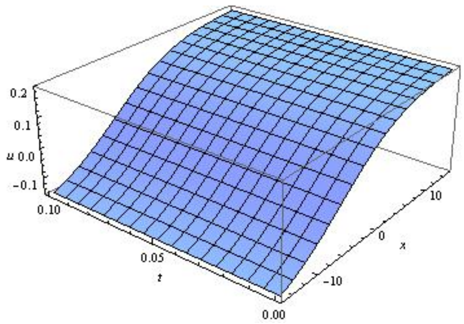

Fig. 3. Approx. soln $\alpha = 1,\ k = 0.1$

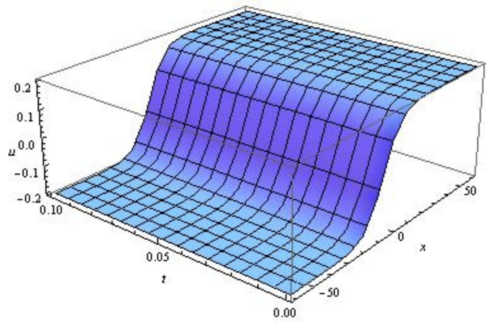

Fig. 4. Approx. soln $\alpha = 0.5,\ k = 0.1$

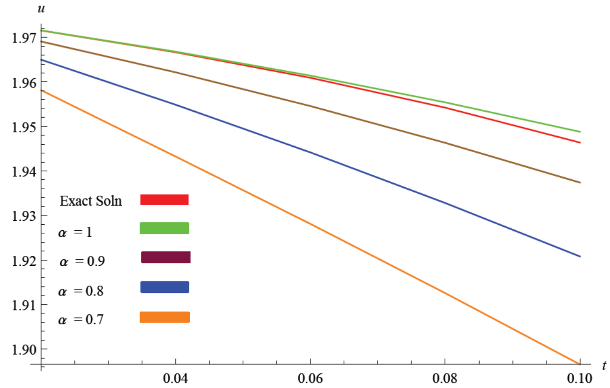

Fig. 5. The curves show the comparison between exact solution and approximate solution obtained by NIM for different values of α **at** $x = 2$

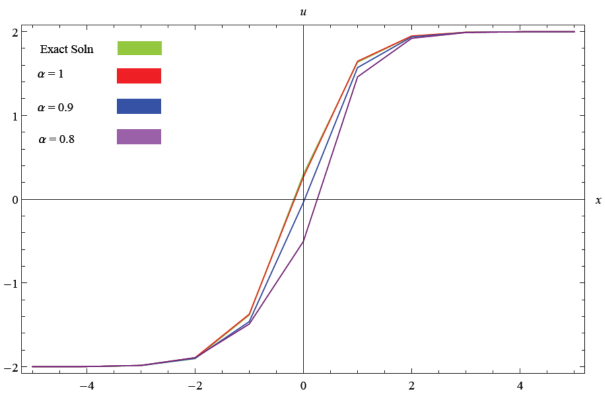

Fig. 6. The curves show the comparison between exact solution and approximate solution obtained by NIM for different values of α **for** $x = -5\ to\ 5$

Table 1. The numerical results for comparison between the exact solution with three term approximations obtained by NIM of (1.1) for $\alpha = 1$ *and* 0.9, and Absolute errors for difference between exact solution and approximate solution

x	t	Exact Solution	NIM solution for $\alpha = 1$	NIM solution for $\alpha = 0.9$	Absolute error $\lvert u - u_3 \rvert$ for $\alpha = 1$
0	0.02	0.8752770400	0.8751740800	0.8040024121	1.02964×10^{-4}
	0.04	0.7415238133	0.7403852800	0.6144304104	1.13853×10^{-3}
	0.06	0.5996114781	0.5947004800	0.4145303303	4.90100×10^{-3}
	0.08	0.4507396834	0.4365644800	0.1986204323	1.41752×10^{-2}
	0.1	0.2964129769	0.2637999999	-0.0385524871	3.26130×10^{-2}
2	0.02	1.9715457264	1.9715622197	1.9690711479	1.64933×10^{-4}
	0.04	1.9666497642	1.9667869637	1.9621408451	1.37199×10^{-4}
	0.06	1.9609196716	1.9614015085	1.9545640642	4.81837×10^{-4}
	0.08	1.9542164228	1.9554057517	1.9463143350	1.18933×10^{-3}
	0.1	1.9463789652	1.9487995503	1.9373934510	2.42059×10^{-3}
4	0.02	1.9994751768	1.9994754944	1.9994292275	3.17600×10^{-7}
	0.04	1.9993841283	1.9993867771	1.9993003705	2.64880×10^{-6}
	0.06	1.9992772871	1.9992866144	1.9991593325	9.32730×10^{-6}
	0.08	1.9991519152	1.9991750063	1.9990056430	2.30911×10^{-5}
	0.1	1.9990047999	1.9990519529	1.9988393536	4.71530×10^{-5}
6	0.02	1.9999903862	1.9999903921	1.9999895444	5.90000×10^{-9}
	0.04	1.9999887182	1.9999887667	1.9999871837	4.85000×10^{-8}
	0.06	1.9999867607	1.9999869317	1.9999845997	1.71000×10^{-7}
	0.08	1.9999844635	1.9999848869	1.9999817839	4.23400×10^{-7}
	0.1	1.9999817678	1.9999826324	1.9999787372	8.64600×10^{-7}
8	0.02	1.9999998239	1.9999998240	1.9999998085	1.00000×10^{-10}
	0.04	1.9999997933	1.9999997942	1.9999997652	9.00000×10^{-10}
	0.06	1.9999997575	1.9999997606	1.9999997179	3.10000×10^{-9}
	0.08	1.9999997154	1.9999997231	1.9999996663	7.70000×10^{-9}
	0.1	1.9999996660	1.9999996819	1.9999996105	1.59000×10^{-9}

6 Conclusions

In this paper, the fractional complex transform with the help of new iterative method is successfully applied to solve time fractional Sharma−Tasso−Olever equation based on Jumarie's modified Riemann-Liouville derivatives and we have observed that the solutions obtained by using this method converges very rapidly to the exact solutions in only second order approximations. From the numerical results we observe that the present technique is straightforward and efficient. This combination of fractional complex transform with new iterative method is very simple, accurate and reliable method for finding approximate solutions of many fractional physical models arising in science and engineering.

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Competing Interests

Authors have declared that no competing interests exist.

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