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Mathematical Model of Blood Flow in Small Blood Vessel in the Presence of Magnetic Field

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Abstract

A mathematical model for blood flow in the small blood vessel in the presence of magnetic field is presented in this paper. We have modeled the two phase model for the blood flow consists of a central core of suspended erythrocytes and cell-free layer surrounding the core. The system of differential equations has been solved analytically. We have obtained the result for velocity, flow rate and effective viscosity in presence of peripheral layer and magnetic field .All the result has been obtained and discussed through graphs.

Keywords: Blood, Plasma, Magnetic Field, Effective Viscosity, Peripheral Layer

1. Introduction

Blood shows anomalous viscous properties. The anomalous behavior of blood is principally due to the suspension of particles in plasma. The plasma solution in the blood obeys the linear Newtonian model for viscosity [1]. However, blood as a whole is often considered as non-Newtonian fluid, particularly when the characteristic dimension of the flow is close to the cell dimension. The experimental observations and theoretical analysis of blood flow are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in animal or human physiology [2]. The flow of blood through small diameter tubes is of physiological and clinical importance. Due to its complexity and anomalous behaviour, it is very difficult to analyze it. The two types of anomaly are due to low shear and high shear effects [3]. When blood flows through larger diameter arteries at high shear rates, it behaves like a Newtonian fluid. The apparent viscosity of blood decreases with decreasing blood vessel diameter, when measurements are made in capillaries of diameter less than 300 μm [4].

Pries *et al.* [5] studied the effect of the tube diameter and the hematocrit ratio on the blood viscosity and found that for tube diameters greater than 1 mm, the blood viscosity is independent of the diameter while for tube diameter less than 1 mm, the blood viscosity is strongly dependent on the tube diameter. They also reported the viscosity increases non-linearly with the hematocrit. Bugliarello and Sevilla [6], Cokelet [7] and [8-10] have reported that for blood flowing through narrow blood vessels, there is a peripheral layer of plasma and a core region of suspension of all the erythrocytes.

Also the red blood cell is major bio-magnetic substance and the blood flow may be influenced by the magnetic field [11]. The effect of magnetic field on blood flow has been analyzed theoretically by treating blood as an electrically conductive fluid [12]. Assuming blood as a magnetic fluid, it may be possible to control blood pressure and its flow behavior by using an appropriate magnetic field. Hence such studies have potential for therapeutic use in the diseases of heart, blood and blood vessels.

Most of the model [13-17] on blood flow deal with one phase model. However, in view of the fact that blood is a suspension, a two-phase model appears to be more appropriate. Wagh and Wagh [18] have used [19] a model of dusty gas to study the effect of the magnetic nature of red blood cells of the flow of blood. The reason is, blood is a liquid suspension having mass and volume concentrations roughly the same, however, for dusty gas the mass and bulk concentrations are quite different [20], Nayfeh's [21] two phase model seems to be more suitable for blood flow.

In view of the above mentioned fact, we have considered the two phase model consisting of central core of suspended erythrocyte and cell free layer surrounding the core in the presence of magnetic field.

2. Mathematical Analysis

We have considered a two layer model (**Figure 1**) for the blood flow within cylindrical vessel of radius R consisting of central core radius R_1, which contains an erythrocyte suspension of uniform hematocrit and a cell free layer outside the core containing plasma. We have taken some assumptions for formulating the mathematical model.

Blood is considered as viscous, incompressible and electrically conducting fluid. Fluid flow is steady and laminar. Magnetic field is constant in transverse direction.

2.1. Governing Equations

Introducing the assumptions mentioned above the governing equation for the fluid flow are given as.

$$\frac{\partial P'}{\partial z'} = \frac{\mu_0}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u_0'}{\partial r'}\right), R_1 < r' < R_0 \tag{1}$$

$$0 = (1-\phi)\left[-\frac{\partial P'}{\partial z'} + \mu_s(\phi)\left(\frac{\partial^2 u_f'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u_f'}{\partial r'}\right)\right] + \phi F_d\left(u_p' - u_f'\right) \tag{2}$$

$$0 = \phi\left[-\frac{\partial P'}{\partial z'} - F_d\left(u_p' - u_f'\right) + k_0 M_p \frac{dH'}{dz'}\right] \tag{3}$$

Where r', z' are radial and axial co-ordinate, u_f and u_p are the velocities of the fluid (plasma) and particles (red cell), ϕ is the volume fraction of the red cells, $\mu_s(\phi)$ is the suspension viscosity, F_d is the drag coefficient of interaction for the force exerted by one phase on the other, k_0 is the magnetic permeability, M_p is the magnetization of red cells and $\frac{dH'}{dz'}$ is the magnetic field gradient. The expression of the drag coefficient is given by:

$$F_d = \frac{9}{2}\frac{\mu_f}{R^2}\lambda(\varphi), \text{ where } \lambda(\varphi) = \frac{4 + 3\left(8\varphi - 3\varphi^2\right)^{\frac{1}{2}} + 3\varphi}{(2-3\varphi)^2}$$

Where μ_f is the constant fluid viscosity.

The viscosity of the suspension is given by an empirical relation.

$$\mu_s = \frac{\mu_f}{(1-\beta\varphi)}, \text{ when}$$

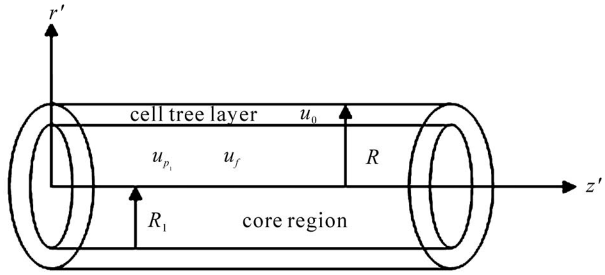

Figure 1. Flow geometry of blood in small vessels.

$$\beta = 0.07\exp\left\{2.49\varphi + \frac{1107}{T}\exp(-1.69\varphi)\right\}$$

2.2. Boundary Conditions

$$\left.\begin{array}{lll} u_0' = 0 & \text{at} & r' = R \\ \dfrac{\partial u_p'}{\partial r'} = \dfrac{\partial u_f'}{\partial r'} = 0 & \text{at} & r' = 0 \\ \left.\begin{array}{c} u_f' = u_0' \\ \text{and} \\ \tau_f' = \tau_0' \end{array}\right\} & \text{at} & r' = R_1' \end{array}\right\} \tag{4}$$

Where $\tau_0' = \mu_0\frac{\partial u_0'}{\partial r'}$ and $\tau_0' = (1-\phi)\mu_s(\phi)\frac{\partial u_f'}{\partial r'}$

2.3. Solution of the Problem

Introducing the following non-dimensional scheme.

$$\left.\begin{array}{llll} r = \dfrac{r'}{R}, & z = \dfrac{z'}{z_0}, & R_1 = \dfrac{R_1'}{R}, & P = \dfrac{P'}{\rho U_0^2} \\ u_f = \dfrac{u_f'}{U_0}, & u_p = \dfrac{u_p'}{U_0}, & \mu = \dfrac{\mu_0}{\mu_s}, & e = \dfrac{\mu_0}{R' z_0} \\ R_e = \dfrac{U_0 R\rho}{\mu_0}, & u_0 = \dfrac{u_0'}{R'}, & H = \dfrac{H'}{\rho U_0^2}, & \varepsilon_0 = \dfrac{R}{z_0} \end{array}\right\} \tag{5}$$

The expression for the velocities u_0, u_f, and u_p, obtained as the solution of Equations (1)-(3), subjected to the boundary conditions are given as:

$$u_0 = \frac{R_e}{4}\varepsilon\frac{\partial P}{\partial z}\left(R_1^2 - 1\right) \tag{6}$$

$$u_f = \frac{R_e}{4}\frac{\partial P}{\partial z}\left[\frac{1}{(1-\phi)\mu_s}\left\{e\phi R^2 + (1-\phi)\varepsilon\right\}\left(r^2 - R_1^2\right) + \varepsilon\left(R_1^2 - 1\right)\right] - \frac{\phi R_e R^2 e M_p k_0}{4(1-\phi)\mu_s}\cdot\frac{dH}{dz}\left(r^2 - R_1^2\right) \tag{7}$$

$$u_p = \frac{R_e}{4}\frac{\partial P}{\partial z}\left[\frac{1}{(1-\phi)\mu_s}\left\{e\phi R^2+(1-\phi)\varepsilon\right\}\left(r^2-R_1^2\right)+\varepsilon\left(R_1^2-1\right)-\frac{4e}{F_d}\right]+R_eM_pk_0\frac{dH}{dz}\left[\frac{e}{F_d}-\frac{\phi R^2 e}{4(1-\phi)\mu_s}\left(r^2-R_1^2\right)\right] \tag{8}$$

The flow flux (volumetric flow rate) is now calculated as

$$Q = Q_0 + Q_f + Q_p \tag{9}$$

Using Equations (6)-(8) in the Equation (9) the expression for flow rate is obtained as:

$$\begin{aligned} Q = {} & \frac{\pi R^4}{8\mu_S}\frac{\partial P}{\partial z}U_0R_e\mu_s\varepsilon\left(1-R_1^2\right)\left(R^2+R_1^2R^2-2\right)+\frac{2R_1^4U_0R_e}{(1-\phi)}\cdot\left\{\left(e\phi R^2+(1-\phi)\varepsilon\right)\left(R^2-2\right)+\frac{2\varepsilon}{R_1^2}\left(R_1^2-1\right)\right\} \\ & +\frac{4\phi U_0R_e\mu_s e}{F_d}+\frac{8\phi R_1^4U_0R_eM_pk_0e\mu_s}{\left(\frac{\partial P}{\partial z}\right)}\cdot\frac{dH}{dZ}\left\{\frac{1}{F_dR_1^2}-\left(R^4-2R^2\right)\left(1+\frac{4\phi}{(1-\phi)\mu_s}\right)\right\} \end{aligned} \tag{10}$$

Using the fact that total flux is equal to the sum of the fluxes across the two regions (peripheral and core) determines the relation.

$$R_1 = \alpha R \tag{11}$$

Using the relation (10) and (11), the expression for the effective (apparent) viscosity is given by:

$$\begin{aligned} \mu_e = \mu_s \Bigg[& U_0R_e\mu_s\varepsilon\left(1-\alpha^2R^2\right)\left(R^2+R^4\alpha-2\right)+\frac{2\alpha^4R^4U_0R_e}{(1-\phi)}\left\{\begin{array}{l}\left(eR^2\phi+(1-\phi)\varepsilon\right)\left(R^2-2\right) \\ +\dfrac{2\varepsilon\left(\alpha^2R^2-1\right)}{\alpha^2R^2}\end{array}\right\} \\ & -\frac{4\phi U_0R_e\mu_s e}{F_d}+\frac{8\phi\alpha^4R^4U_0R_eM_pk_0e\mu_s}{\left(\frac{\partial P}{\partial z}\right)}\cdot\frac{dH}{dz}\left\{\frac{1}{F_d\alpha^2R^2}-\left(R^4-2R^2\right)\left(1+\frac{4\phi}{(1-\phi)\mu_s}\right)\right\}\Bigg]^{-1} \end{aligned} \tag{12}$$

3. Results and Discussions

To have a quantitave estimate of the various parameters involved, particularly the hematocrit (ϕ) and magnetic field gradient $\left(H_Z = \frac{\partial H}{\partial z}\right)$, some of the results is displayed graphically in **Figures 2-8**.

The variation of effective viscosity (μ_e) for different values of magnetic field gradient (H_z) is shown in **Figure 2** and for different values of hematocrit (ϕ) is shown in **Figure 3**. The effective viscosity increases with increasing value of hematocrit. Also, the effective viscosity increases with the increase of the magnetic field gradient. The major mechanism of the influence of a static magnetic field on blood flow viscosity is based on the interaction between the induced magnetic moment on the RBC and the external static magnetic field. This property in a static magnetic field increases the friction of the flowing blood, because the anisotropic orientation of the RBC in the static magnetic field distribus the rolling of the cell in flowing blood, and so the blood viscosity increases [22].

Figure 4 shows the variation of flow rate (Q) with hematocrit of blood (ϕ) for different values of magnetic field gradient (H_z). It is clear from the figure that the flow rate decreases slowly with the increase of the

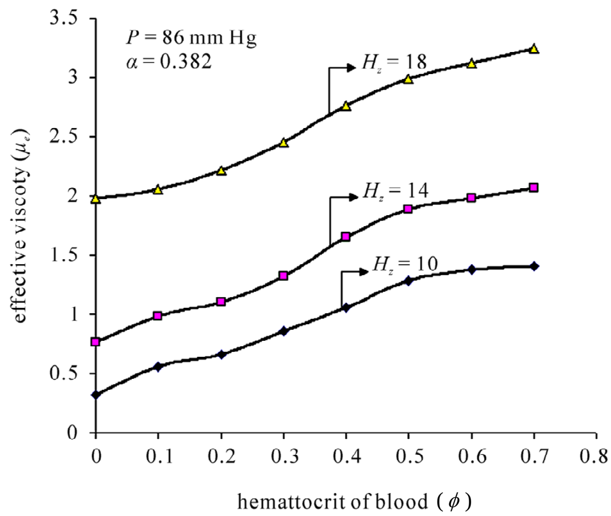

Figure 2. Variation of effective viscosity (μ_e) with hematocrit of blood (ϕ) for different values of magnetic field gradient $H_z = (dH/dz)$.

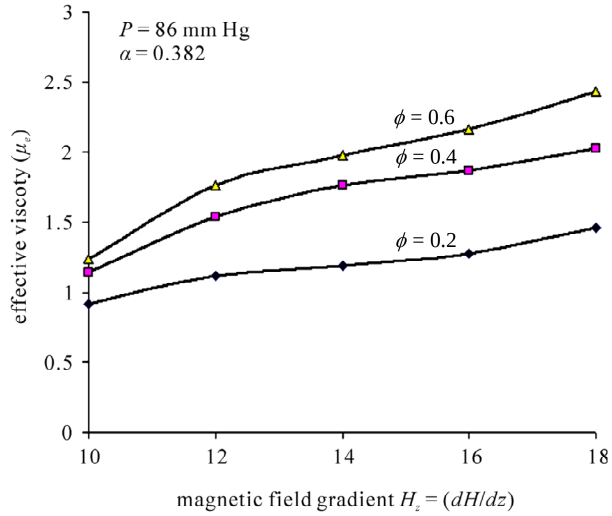

Figure 3. Variation of effective viscosity (μ_e) with magnetic field gradient $H_z = (dH/dz)$ for different values hematocrit (ϕ).

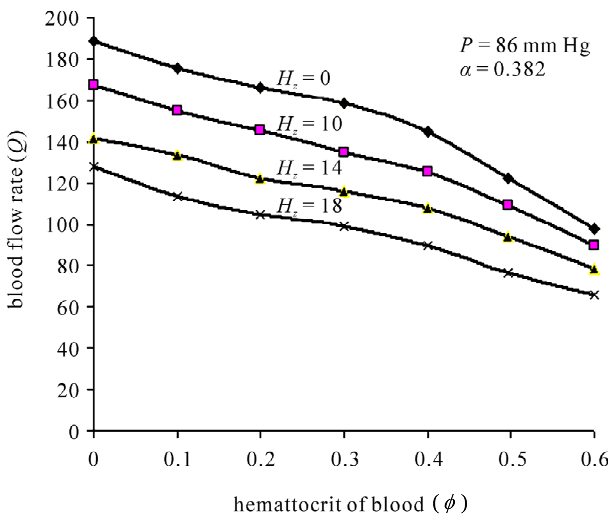

Figure 4. Variation of blood flow rate (Q) with hematocrit of blood (ϕ) for different values of magnetic field gradient $H_z = (dH/dz)$.

hematocrit and decrease with increasing values of the magnetic field gradient. It may be observed that the flow rate is significantly influenced by the magnetic field gradient, the magnetic nature of the fluid and hematocrit of blood (ϕ).

The variation of axial velocities profiles u_f and u_p for both phase (plasma and erythrocyte) with radial axis (r) for different values of hematocrit (ϕ) are plotted in **Figures 5** and **7**. It has been observed that the u_f and u_p decrease with increase of the hematocrit (ϕ) for constant magnetic field gradient (H_z) and other parameter are keep constant. The effect of hematocrit of blood (ϕ) on the velocity is relatively small near the wall. It may be due to the red cell's tendency to accumulate near the tube axis.

Figures 6 and **8**, shows the variation of axial velocites profiles u_f and u_p for both phase (plasma and erythrocyte) with radial axis (r) for different values of magnetic field gradient (H_z). It is clear from the figure that u_f and u_p decrease with increase of the magnetic field gradient. Thus, it is of importance to note that though the suspending fluid is non magnetic the magnetic field gradient influence its velocity.

All these results of the present study have been compared with already existing results obtained in the theoretical study of [2], [13] and [21].

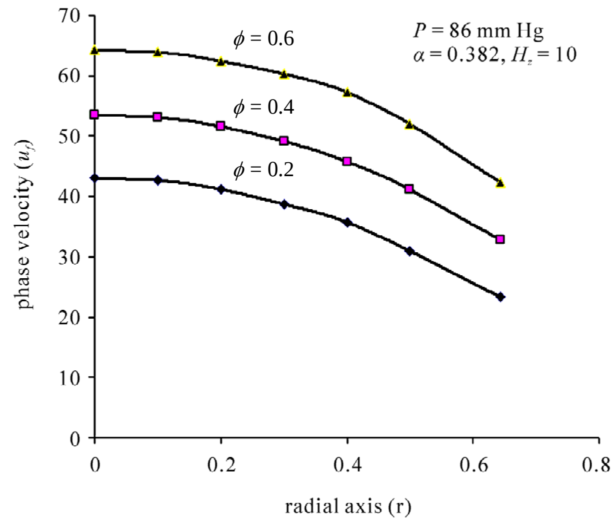

Figure 5. Variation of phase velocity (u_f) with radial axis for different values of hematocrit of blood (ϕ).

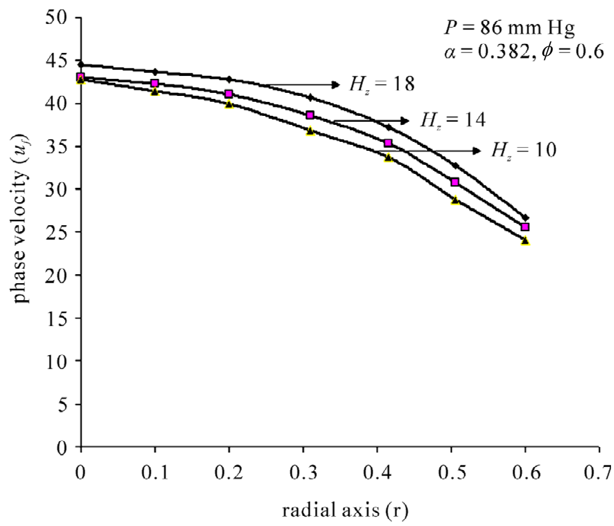

Figure 6. Variation of phase velocity (u_f) with radial axis for different values of magnetic field gradient $H_z = (dH/dz)$.

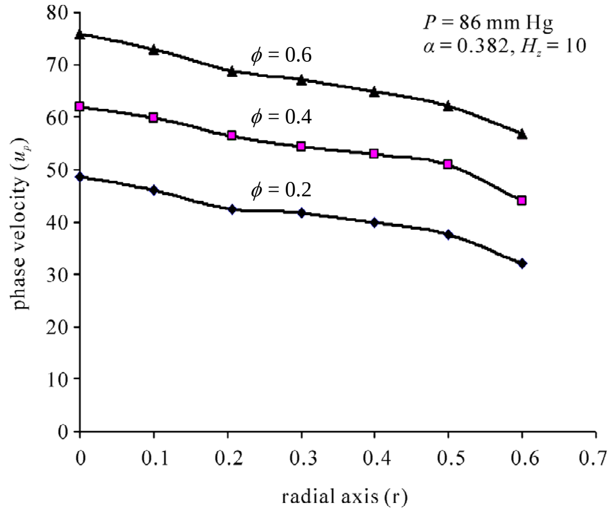

Figure 7. Variation of pluge velocity (u_p) with radial axis for different values of hematocrit of blood (ϕ).

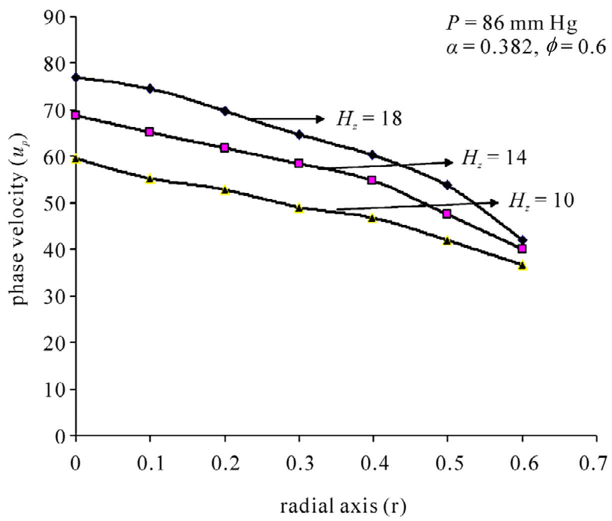

Figure 8. Variation of pluge velocity (u_p) with radial axis for different values of magnetic field gradient $H_z = (dH/dz)$.

4. Conclusions

This study brings out many interesting fluid mechanical phenomena due to the magnetic field and presence of the peripheral layer. Blood has been modeled as two-fluid model with the core region of suspension of all the erythrocytes and the plasma in the peripheral region as a Newtonian fluid. It is noted that the velocity and flow rate decreases, while the effective viscosity increases with magnetic field and hematocrit .

It is clear from the above discussion that magnetic field affects largely on the axial flow velocities of blood and effective viscosity. So, by taking appropriate values of magnetic field we may regulate the axial velocities and effective viscosity.

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