Journal of Scientific Research & Reports
6(7): 490-503, 2015, Article no.JSRR.2015.174
ISSN: 2320-0227

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Almost Periodic Solution of a Discrete Multispecies Type Competition-predator System

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Article Information

DOI: 10.9734/JSRR/2015/16779
Editor(s):
(1) Narcisa C. Apreutesei, Technical University of Iasi, Romania.
Reviewers:
(1) Anonymous, Egypt.
(2) Anonymous, Malaysia.
(3) Anonymous, India.
(4) Anonymous, China.
(5) Muahmmad Altaf Khan, Abdul Wali Khan University Mardan, Pakistan.
Complete Peer review History:
http://www.sciencedomain.org/review-history.php?iid=968&id=22&aid=8615

Original Research Article

Received: 13 February 2015
Accepted: 20 March 2015
Published: 30 March 2015

Abstract

In this paper, we consider a discrete multispecies Gilpin-Ayala type competition-predator system. Firstly, permanence of the system is studied. Assume that the coefficients in the system are almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive by the almost periodicity. Two examples together with numerical simulation indicate the feasibility of the main results.

Keywords: Almost periodic solution; discrete; competition-predator system; permanence; global attractivity.

2010 Mathematics Subject Classification: 39A11

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1 Introduction

Recently, more and more authors have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, also, since discrete time models can provide efficient computational models of continuous models for numerical simulations, it is reasonable to study discrete time population models governed by difference equations.

In 2007, Chen et al. [1] had investigated the dynamic behavior of the following discrete n-species Gilpin-Ayala competition model

$$x_i(k+1) = x_i(k)\exp\left\{b_i(k) - \sum_{j=1}^{n} a_{ij}(k)(x_j(k))^{\theta_{ij}}\right\}, \tag{1.1}$$

where $i = 1, 2, \cdots, n$; $x_i(k)$ is the density of competition species i at k-th generation. $a_{ij}(k)$ measures the intensity of intraspecific competition or interspecific action of competition species, respectively. $b_i(k)$ representing the intrinsic growth rate of the competition species x_i. θ_{ij} are positive constants. $b_i(k), a_{ij}(k), i, j = 1, 2, \cdots, n$ are all positive sequences bounded above and below by positive constants. Obviously, when $\theta_{ij} \equiv 1$, system (1.1) reduces to the traditional discrete multispecies Lotka-Volterra competition model

$$x_i(k+1) = x_i(k)\exp\left\{b_i(k) - \sum_{j=1}^{n} a_{ij}(k)x_j(k)\right\}.$$

The next goal of Ref. [1] is to investigate the dynamic behavior of the following discrete $n+m$-species Gilpin-Ayala type competition-predator model

$$\begin{aligned} x_i(k+1) &= x_i(k)\exp\Big[b_i(k) - \sum_{l=1}^{n} a_{il}(k)x_l^{\alpha_{il}}(k) - \sum_{l=1}^{n} c_{il}(k)x_i^{\alpha_{ii}}(k)x_l^{\alpha_{il}}(k) \\ &\qquad - \sum_{l=1}^{m} d_{il}(k)y_l^{\beta_{il}}(k)\Big], \\ y_j(k+1) &= y_j(k)\exp\Big[-r_j(k) + \sum_{l=1}^{n} e_{jl}(k)x_l^{\delta_{jl}}(k) - \sum_{l=1}^{m} f_{jl}(k)y_j^{\eta_{jj}}(k)y_l^{\eta_{jl}}(k) \\ &\qquad - \sum_{l=1}^{m} g_{jl}(k)y_l^{\eta_{jl}}(k)\Big], \end{aligned} \tag{1.1}$$

where $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, m$; $x_i(k)$ is the density of prey species i at k-th generation, $y_j(k)$ is the density of predator species j at k-th generation. α_{il}, β_{il}, δ_{jl}, and η_{jl} are all positive constants; $a_{il}(k)$, $c_{il}(k)$ and $f_{jl}(k)$, $g_{jl}(k)$) measures the intensity of intraspecific competition or interspecific action of prey species and predator species, respectively. $b_i(k)$ representing the intrinsic growth rate of the prey species x_i; $r_j(k)$) representing the death rate of the predator species y_j.

For general nonautonomous case, sufficient conditions which ensure the permanence and the global stability of system (1.1) and (1.2) are obtained; For periodic case, sufficient conditions which ensure the existence of a unique globally stable positive periodic solution of system (1.1) and (1.2) are obtained.

Notice that the investigation of almost periodic solutions for difference equations is one of most important topics in the qualitative theory of difference equations due to the applications in biology, ecology, neural network, and so forth (see [2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12],[13],[14],[15] and the references cited therein). Li and Chen [2] studied an almost periodic discrete logistic equation

$$x(n+1) = x(n)\exp\left(r(n)\big(1-\frac{x(n)}{K(n)}\big)\right).$$

Sufficient conditions are obtained for the existence of a unique almost periodic solution which is globally attractive. Wang and Liu [3] studied a discrete Lotka-Volterra competitive system

$$\begin{cases} x_1(n+1) = x_1(n)\exp\left[r_1(n) - a_1(n)x_1(n) - \dfrac{c_2(n)x_2(n)}{1+x_2(n)}\right], \\ x_2(n+1) = x_2(n)\exp\left[r_2(n) - a_2(n)x_2(n) - \dfrac{c_1(n)x_1(n)}{1+x_1(n)}\right], \quad n = 0,1,2,\cdots. \end{cases}$$

With the help of the methods of the Lyapunov function, some analysis techniques, and preliminary lemmas, they establish a criterion for the existence, uniqueness and uniformly asymptotic stability of positive almost periodic solution of the system. Li and Chen [4] had studied extinction and almost periodic solutions of system (1.1). Assume that the coefficients in system (1.1) are almost periodic sequences, they obtained that r of the species in system (1.1) are permanent and stabilize at a unique strictly positive almost periodic solution of the corresponding subsystem, which is globally attractive, while the remaining n-r species are driven to extinction. However, few work has been done previously on an almost periodic version which is corresponding to system (1.2). Then, we will further investigate the global stability of almost periodic solution of system (1.2) with stimulation from the works of [8].

Denote as Z and Z^+ the set of integers and the set of nonnegative integers, respectively. For any bounded sequence $\{g(n)\}$ defined on Z, define $g^u = \sup_{n\in Z} g(n), g^l = \inf_{n\in Z} g(n)$.

Throughout this paper, we assume that:

(H1) $b_i(k)$, $a_{il}(k)$, $c_{il}(k)$, $d_{il}(k)$, $r_j(k)$, $e_{jl}(k)$, $f_{jl}(k)$ and $g_{jl}(k)$ are bounded nonnegative almost periodic sequences such that

$$0 < b_i^l \le b_i(k) \le b_i^u,\ 0 < a_{il}^l \le a_{il}(k) \le a_{il}^u,\ 0 < c_{il}^l \le c_{il}(k) \le c_{il}^u,\ 0 < e_{il}^l \le e_{il}(k) \le e_{il}^u,$$

$l = 1,2,\cdots,n,$

$$0 < r_j^l \le r_j(k) \le r_j^u,\ 0 < d_{jl}^l \le d_{jl}(k) \le d_{jl}^u,\ 0 < f_{jl}^l \le f_{jl}(k) \le f_{jl}^u,\ 0 < g_{jl}^l \le g_{jl}(k) \le g_{jl}^u,$$

$l = 1,2,\cdots,m,$
$i = 1,2,\cdots,n,\ j = 1,2,\cdots,m.$

From the point of view of biology, in the sequel, we assume that $\mathbf{x}(0) = (x_1(0), x_2(0), \cdots, x_n(0), y_1(0), y_2(0), \cdots, y_m(0)) > \mathbf{0}$. Then it is easy to see that, for given $\mathbf{x}(0) > \mathbf{0}$, the system (1.1) has a positive sequence solution $\mathbf{x}(k) = (x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))(k \in Z^+)$ passing through $\mathbf{x}(0)$.

The remaining part of this paper is organized as follows: In Section 2, we will introduce some definitions and several useful lemmas. In Section 3, we present the permanence results for system (1.2). In Section 4, we establish the sufficient conditions for the existence of a unique globally attractive almost periodic solution of system (1.2). The main results are illustrated by two examples with numerical simulation in Section 5. Finally, the conclusion ends with brief remarks in the last section.

2 Preliminaries

Firstly, we give the definitions of the terminologies involved.

Definition 2.1 [16] A sequence $x : Z \to R$ is called an almost periodic sequence if the ε-translation set of x

$$E\{\varepsilon, x\} = \{\tau \in Z :| x(n+\tau) - x(n) |< \varepsilon, \forall n \in Z\}$$

is a relatively dense set in Z for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains an integer $\tau \in E\{\varepsilon, x\}$ with

$$| x(n+\tau) - x(n) |< \varepsilon, \quad \forall n \in Z.$$

τ is called an ε-translation number of $x(n)$.

Definition 2.2 [17] Let D be an open subset of R^m, $f : Z \times D \to R^m$. $f(n, x)$ is said to be almost periodic in n uniformly for $x \in D$ if for any $\varepsilon > 0$ and any compact set $S \subset D$, there exists a positive integer $l = l(\varepsilon, S)$ such that any interval of length l contains an integer τ for which

$$|f(n+\tau, x) - f(n, x)| < \varepsilon, \quad \forall (n, x) \in Z \times S.$$

τ is called an ε-translation number of $f(n, x)$.

Definition 2.3 [18] The hull of f, denoted by $H(f)$, is defined by

$$H(f) = \{g(n, x) : \lim_{k\to\infty} f(n+\tau_k, x) = g(n, x) \text{ uniformly on } Z \times S\},$$

for some sequence $\{\tau_k\}$, where S is any compact set in D.

Definition 2.4 [19] A sequence $x : Z^+ \to R$ is called an asymptotically almost periodic sequence if

$$x(n) = p(n) + q(n), \ \ \forall n \in Z^+,$$

where $p(n)$ is an almost periodic sequence and $\lim\limits_{n\to+\infty} q(n) = 0$.

Lemma 2.5 [20] $\{x(n)\}$ is an almost periodic sequence if and only if for any integer sequence $\{k_i'\}$, there exists a subsequence $\{k_i\} \subset \{k_i'\}$ such that the sequence $\{x(n+k_i)\}$ converges uniformly for all $n \in Z$ as $i \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.

Lemma 2.6 [19] $\{x(n)\}$ is an asymptotically almost periodic sequence if and only if, for any sequence $m_i \subset Z$ satisfying $m_i > 0$ and $m_i \to \infty$ as $i \to \infty$ there exists a subsequence $\{m_{i_k}\} \subset \{m_i\}$ such that the sequence $\{x(n+m_{i_k})\}$ converges uniformly for all $n \in Z^+$ as $k \to \infty$.

3 Permanence

In this section, we establish a permanence result for system (1.2), which can be given in [1].

Proposition 3.1 Assume that (H1) holds. Then any positive solution $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfies

$$\limsup_{k\to+\infty} x_i(k) \le M_i, \tag{3.1}$$

where

$$M_i = \left(\frac{1}{\alpha_{ii} a_{ii}^l}\right)^{\frac{1}{\alpha_{ii}}} \exp\left\{b_i^u - \frac{1}{\alpha_{ii}}\right\}, \ \ i = 1, 2, \cdots, n.$$

Proposition 3.2 Assume that (H1) and

(H2) $-r_j^l + \sum\limits_{j=1}^{n} e_{jl}^u (M_l)^{\delta_{jl}} > 0$

hold for all $i = 1, 2, \cdots, n$, where $M_i,\ i = 1, 2, \cdots, n$ are defined by (3.1). Then for every solution $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfies

$$\limsup_{k\to+\infty} y_j(k) \le N_j, \tag{3.2}$$

where

$$N_j = \Big(\frac{1}{\eta_{jj} g_{jj}^l}\Big)^{\dfrac{1}{\eta_{jj}}} \exp\Big\{ -r_j^l + \sum_{l=1}^{n} e_{jl}^u (M_l)^{\delta_{jl}} - \frac{1}{\eta_{jj}} \Big\},\ \ j = 1, 2, \cdots, m.$$

Proposition 3.3 Assume that (H1) and (H2) hold, assume further that

(H3) $b_i^l - \sum\limits_{l=1,l\neq i}^{n} a_{il}^u (M_l)^{\alpha_{il}} - \sum\limits_{l=1}^{m} d_{il}^u (N_l)^{\beta_{il}} > 0$

hold for all $i = 1, 2, \cdots, n$, where $M_i,\ i = 1, 2, \cdots, n$ and $N_j,\ j = 1, 2, \cdots, m$ are defined by (3.1) and (3.2). Then for every solution $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfies

$$\liminf_{k\to+\infty} x_i(k) \ge m_i, \tag{3.3}$$

where

$$\begin{aligned} m_i &= A_i \exp\{B_i\}, \\ A_i &= \left(\frac{b_i^l - \sum\limits_{l=1,l\neq i}^{u} a_{il}^u (M_l)^{\alpha_{il}} - \sum\limits_{l=1}^{m} d_{il}^u (N_l)^{\beta_{il}}}{a_{ii}^u + \sum\limits_{l=1}^{n} c_{il}^u (M_l)^{\alpha_{il}}} \right)^{\dfrac{1}{\alpha_{ii}}}, \\ B_i &= b_i^l - \sum_{l=1}^{n} a_{il}^u (M_l)^{\alpha_{il}} - \sum_{l=1}^{n} c_{il}^u (M_i)^{\alpha_{ii}} (M_l)^{\alpha_{il}} - \sum_{l=1}^{m} d_{il}^u (N_l)^{\beta_{il}}, \end{aligned}$$

$i = 1, 2, \cdots, n.$

Proposition 3.4 Assume that (H1)-(H3) hold; assume further that

(H4) $-r_j^u + \sum\limits_{l=1}^{n} e_{jl}^l (m_l)^{\delta_{jl}} - \sum\limits_{l=1,l\neq j}^{m} g_{jl}^u (N_l)^{\eta_{jl}} > 0$

hold for all $j = 1, 2, \cdots, m$, where $N_j,\ j = 1, 2, \cdots, m$ and $m_i,\ i = 1, 2, \cdots, n$ are defined by (3.2) and (3.3). Then for every solution $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfies

$$\liminf_{k\to+\infty} y_j(k) \ge n_j,$$

where

$$\begin{aligned} n_j &= C_j \exp\{D_j\}, \\ C_j &= \left(\frac{-r_j^u + \sum\limits_{l=1}^{u} e_{jl}^l (m_l)^{\delta_{jl}} - \sum\limits_{l=1,l\neq j}^{m} g_{jl}^u (N_l)^{\eta_{jl}}}{g_{jj}^u + \sum\limits_{l=1}^{m} f_{jl}^u (M_l)^{\eta_{jl}}} \right)^{\dfrac{1}{\eta_{jj}}}, \\ D_j &= -r_j^u + \sum_{l=1}^{n} e_{jl}^u (m_l)^{\delta_{jl}} - \sum_{l=1}^{m} f_{jl}^u (N_j)^{\eta_{jj}} (M_l)^{\eta_{jl}} - \sum_{l=1}^{m} g_{jl}^u (N_l)^{\eta_{jl}}, \end{aligned}$$

$j = 1, 2, \cdots, m$.

As the direct corollary of Proposition 3.1-3.4, we have

Theorem 3.5 Assume that (H1)-(H4) hold, then system (1.2) is permanent.

The next result tells us that there exist solutions of system (1.2) totally in the interval of Theorem 3.5. We denote by Ω the set of all solutions $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfying $m_i \leq x_i(k) \leq M_i (i = 1, 2, \cdots, n)$ and $n_j \leq y_j(k) \leq N_j (j = 1, 2, \cdots, m)$ for all $k \in Z^+$.

Proposition 3.6 Assume that (H1)-(H4) hold. Then $\Omega \neq \Phi$.

Proof. By the almost periodicity of $b_i(k), a_{il}(k), c_{il}(k), d_{il}(k), r_j(k), e_{jl}(k), f_{jl}(k)$ and $g_{jl}(k)$, there exists an integer valued sequence $\{\delta_p\}$ with $\delta_p \to +\infty$ as $p \to +\infty$ such that

$$b_i(k+\delta_p) \to b_i(k), \;\; a_{il}(k+\delta_p) \to a_{il}(k), \;\; c_{il}(k+\delta_p) \to c_{il}(k), \;\; d_{il}(k+\delta_p) \to d_{il}(k),$$

$$r_j(k+\delta_p) \to r_j(k), \;\; e_{jl}(k+\delta_p) \to e_{jl}(k), \;\; f_{jl}(k+\delta_p) \to f_{jl}(k), \;\; g_{jl}(k+\delta_p) \to g_{jl}(k), \;\; \text{as} \;\; p \to +\infty.$$

Let ε be an arbitrary small positive number. It follows from Theorem 3.5 that there exists a positive integer N_0 such that

$$m_i - \varepsilon \leq x_i(k) \leq M_i + \varepsilon, \;\; n_j - \varepsilon \leq y_j(k) \leq N_j + \varepsilon, \;\; k > N_0.$$

Write $x_{ip}(k) = x_i(k+\delta_p)$ and $y_{jp}(k) = y_j(k+\delta_p)$ for $k \geq N_0 - \delta_p$ and $p = 1, 2, \cdots$. For any positive integer q, it is easy to see that there exists two sequences $\{x_{ip}(k) : p \geq q\}$ and $\{y_{jp}(k) : p \geq q\}$ such that the sequences $\{x_{ip}(k)\}$ and $\{y_{jp}(k)\}$ have two subsequences, respectively, denoted by $\{x_{ip}(k)\}$ and $\{y_{jp}(k)\}$ again, converging on any finite interval of **Z** as $p \to +\infty$. Thus we have two sequences $\{\widetilde{x}_i(k)\}$ and $\{\widetilde{y}_j(k)\}$ such that

$$x_{ip}(k) \to \widetilde{x}_i(k), \;\; y_{jp}(k) \to \widetilde{y}_j(k) \text{ for } k \in \mathbf{Z} \text{ as } p \to +\infty.$$

This, combined with

$$\begin{aligned} x_i(k+1+\delta_p) &= x_i(k+\delta_p)\exp\Big[b_i(k+\delta_p) - \sum_{l=1}^{n} a_{il}(k+\delta_p)x_l^{\alpha_{il}}(k+\delta_p) \\ &- \sum_{l=1}^{n} c_{il}(k+\delta_p)x_i^{\alpha_{ii}}(k+\delta_p)x_l^{\alpha_{il}}(k+\delta_p) - \sum_{l=1}^{m} d_{il}(k+\delta_p)y_l^{\beta_{il}}(k+\delta_p)\Big], \\ y_j(k+1+\delta_p) &= y_j(k+\delta_p)\exp\Big[-r_j(k+\delta_p) - \sum_{l=1}^{n} e_{jl}(k+\delta_p)x_l^{\delta_{jl}}(k+\delta_p) \\ &- \sum_{l=1}^{m} f_{jl}(k+\delta_p)y_j^{\eta_{jj}}(k+\delta_p)y_l^{\eta_{jl}}(k+\delta_p) - \sum_{l=1}^{m} g_{jl}(k+\delta_p)y_l^{\eta_{jl}}(k+\delta_p)\Big], \\ & i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m \end{aligned}$$

gives us

$$\begin{aligned} \widetilde{x}_i(k+1) &= \widetilde{x}_i(k)\exp\Big[b_i(k) - \sum_{l=1}^{n} a_{il}(k)\widetilde{x}_l^{\alpha_{il}}(k) - \sum_{l=1}^{n} c_{il}(k)\widetilde{x}_i^{\alpha_{ii}}(k)x_l^{\alpha_{il}}(k) - \sum_{l=1}^{m} d_{il}(k)\widetilde{y}_l^{\beta_{il}}(k)\Big], \\ \widetilde{y}_j(k+1) &= \widetilde{y}_j(k)\exp\Big[-r_j(k) - \sum_{l=1}^{n} e_{jl}(k)\widetilde{x}_l^{\delta_{jl}}(k) - \sum_{l=1}^{m} f_{jl}(k)\widetilde{y}_j^{\eta_{jj}}(k)y_l^{\eta_{jl}}(k) - \sum_{l=1}^{m} g_{jl}(k)\widetilde{y}_l^{\eta_{jl}}(k)\Big], \\ & i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m. \end{aligned}$$

We can easily see that $(\widetilde{x}_1(k), \widetilde{x}_2(k), \cdots, \widetilde{x}_n(k), \widetilde{y}_1(k), \widetilde{y}_2(k), \cdots, \widetilde{y}_m(k))$ is a solution of system (1.2) and $m_i - \varepsilon \le \widetilde{x}_i(k) \le M_i + \varepsilon,\ \ n_j - \varepsilon \le \widetilde{y}_j(k) \le N_j + \varepsilon$ for $k \in$ **Z**. Since ε is an arbitrary small positive number, it follows that $m_i \le \widetilde{x}_i(k) \le M_i,\ \ n_j \le \widetilde{y}_j(k) \le N_j$ and hence we complete the proof.

4 Almost Periodic Solution

The main results of this paper concern the existence of a unique globally attractive almost periodic solution of system (1.2). Firstly, we establish a global attractivity result for system (1.2), which can be given in [1].

Theorem 4.1([1]) Assume that (H1)-(H4) and

(H5)
$$\lambda_i = \max\Bigg\{\bigg|1 - \alpha_{ii}\bigg(a^u_{ii}(M_i)^{\alpha_{ii}} + c^u_{ii}(M_i)^{2\alpha_{ii}} + \sum_{l=1}^{n} c^u_{il}(M_i)^{\alpha_{ii}}(M_l)^{\alpha_{il}}\bigg)\bigg|, \bigg|1 - \alpha_{ii}\bigg(a^l_{ii}(m_i)^{\alpha_{ii}} + c^l_{ii}(m_i)^{2\alpha_{ii}} + \sum_{l=1}^{n} c^l_{il}(m_i)^{\alpha_{ii}}(m_l)^{\alpha_{il}}\bigg)\bigg|\Bigg\} + \sum_{l=1, l\ne i}^{n} \alpha_{il}[a^u_{il}(M_l)^{\alpha_{il}} + c^u_{il}(M_i)^{\alpha_{ii}}(M_l)^{\alpha_{il}}] + \sum_{l=1}^{m} \beta_{il} d^u_{il}(N_l)^{\beta_{il}} < 1,\ i = 1, 2, \cdots, n,$$

$$\gamma_j = \max\Bigg\{\bigg|1 - \eta_{jj}\bigg(g^u_{jj}(N_j)^{\eta_{jj}} + f^u_{jj}(N_j)^{2\eta_{jj}} + \sum_{l=1}^{m} f^u_{il}(N_j)^{\eta_{jj}}(N_l)^{\eta_{jl}}\bigg)\bigg|, \bigg|1 - \eta_{jj}\bigg(g^l_{jj}(n_j)^{\eta_{jj}} + f^u_{jj}(n_j)^{2\eta_{jj}} + \sum_{l=1}^{m} f^u_{il}(n_j)^{\eta_{jj}}(n_l)^{\eta_{jl}}\bigg)\bigg|\Bigg\} + \sum_{l=1, l\ne j}^{n} \eta_{jl}[g^u_{jl}(N_l)^{\eta_{jl}} + f^u_{jl}(N_j)^{\eta_{jj}}(N_l)^{\eta_{jl}}] + \sum_{l=1}^{n} \delta_{il} e^u_{jl}(M_l)^{\delta_{jl}} < 1,\ j = 1, 2, \cdots, m,$$

hold. Then for any two positive solutions $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ and $(\widetilde{x}_1(k), \widetilde{x}_2(k), \cdots, \widetilde{x}_n(k), \widetilde{y}_1(k), \widetilde{y}_2(k), \cdots, \widetilde{y}_m(k))$ of system (1.2), we have

$$\lim_{k\to+\infty}(x_i(k) - \widetilde{x}_i(k)) = 0,\ \ i = 1, 2, \cdots, n, \tag{4.1}$$

$$\lim_{k\to+\infty}(y_j(k) - \widetilde{y}_j(k)) = 0,\ \ j = 1, 2, \cdots, m. \tag{4.2}$$

Now, we consider the almost periodic property of system (1.2).

Theorem 4.2 Assume that (H1)-(H5) hold. Then system (1.2) admits a unique almost periodic solution which is globally attractive.

Proof. It follows from Proposition 3.6 that there exists a solution $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ of system (1.2) satisfying $m_i \le x_i(k) \le M_i, n_j \le y_j(k) \le N_j, k \in Z^+$.

Suppose that $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ is any solution of system (1.2), then there exists an integer valued sequence $\{k'_p\}$, $k'_p \to +\infty$ as $p \to +\infty$, such that $(x_1(k + k'_p), x_2(k + k'_p), \cdots, x_n(k + k'_p), y_1(k + k'_p), y_2(k + k'_p), \cdots, y_m(k + k'_p))$ is a solution of the following system

$$x_i(k+1) = x_i(k)\exp\Big[b_i(k+k_p^{'}) - \sum_{l=1}^{n} a_{il}(k+k_p^{'})x_l^{\alpha_{il}}(k) - \sum_{l=1}^{n} c_{il}(k+k_p^{'})x_i^{\alpha_{ii}}(k)x_l^{\alpha_{il}}(k) - \sum_{l=1}^{m} d_{il}(k+k_p^{'})y_l^{\beta_{il}}(k)\Big],$$
$$y_j(k+1) = y_j(k)\exp\Big[-r_j(k+k_p^{'}) - \sum_{l=1}^{n} e_{jl}(k+k_p^{'})x_l^{\delta_{jl}}(k) - \sum_{l=1}^{m} f_{jl}(k+k_p^{'})y_j^{\eta_{jj}}(k)y_l^{\eta_{jl}}(k) - \sum_{l=1}^{m} g_{jl}(k+k_p^{'})y_l^{\eta_{jl}}(k)\Big],$$
$$i = 1,2,\cdots,n,\ j = 1,2,\cdots,m.$$

From above discussion and Theorem 3.5, we have that not only $(x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), y_1(k+k_p^{'}), y_2(k+k_p^{'}), \cdots, y_m(k+k_p^{'}))$ but also $(\Delta x_1(k+k_p^{'}), \Delta x_2(k+k_p^{'}), \cdots, \Delta x_n(k+k_p^{'}), \Delta y_1(k+k_p^{'}), \Delta y_2(k+k_p^{'}), \cdots, \Delta y_m(k+k_p^{'}))$ are uniformly bounded, thus $(x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), y_1(k+k_p^{'}), y_2(k+k_p^{'}), \cdots, y_m(k+k_p^{'}))$ are uniformly bounded and equi-continuous. By Ascoli's theorem[[21]], there exists a uniformly convergent subsequence $(x_1(k+k_p), x_2(k+k_p), \cdots, x_n(k+k_p), y_1(k+k_p), y_2(k+k_p), \cdots, y_m(k+k_p)) \subseteq (x_1(k+k_p^{'}), x_2(k+k_p^{'}), \cdots, x_n(k+k_p^{'}), y_1(k+k_p^{'}), y_2(k+k_p^{'}), \cdots, y_m(k+k_p^{'}))$ such that for any $\varepsilon > 0$, there exists a $k_0(\varepsilon) > 0$ with the property that if $m, n \geq k_0(\varepsilon)$ then

$$|x_i(k+k_m) - x_i(k+k_n)| < \varepsilon,\ \ |y_j(k+k_m) - y_j(k+k_n)| < \varepsilon,$$

which shows from Lemma 2.6 that $(x_1(k), x_2(k), \cdots, x_n(k), y_1(k), y_2(k), \cdots, y_m(k))$ is asymptotically almost periodic sequence. Thus, by Definition 2.4, we can express it as

$$x_i(k) = p_i(k) + q_i(k),\ \ y_j(k) = u_j(k) + v_j(k),$$

$i = 1,2,\cdots,n, j = 1,2,\cdots,m$, where $\{p_i(k)\}$ and $\{u_j(k)\}$ are almost periodic in $k \in Z$ and $q_i(k) \to 0$, $v_j(k) \to 0$ as $k \to +\infty$. In the following we show that $\{(p_1(k), p_2(k), \cdots, p_n(k), u_1(k), u_2(k), \cdots, u_m(k))\}$ is an almost periodic solution of system (1.2).

From the properties of an almost periodic sequence, there exists an integer valued sequence $\{\delta_p\}$, $\delta_p \to +\infty$ as $p \to +\infty$, such that

$$b_i(k+\delta_p) \to b_i(k),\ \ a_{il}(k+\delta_p) \to a_{il}(k),\ \ c_{il}(k+\delta_p) \to c_{il}(k),\ \ d_{il}(k+\delta_p) \to d_{il}(k),$$

$$r_j(k+\delta_p) \to r_j(k),\ \ e_{jl}(k+\delta_p) \to e_{jl}(k),\ \ f_{jl}(k+\delta_p) \to f_{jl}(k),\ \ g_{jl}(k+\delta_p) \to g_{jl}(k),\ \text{ as } p \to +\infty.$$

It is easy to know that $x_i(k+\delta_p) \to p_i(k)$, $y_j(k+\delta_p) \to u_j(k)$ as $p \to \infty$, then we have

$$\begin{aligned} p_i(k+1) &= \lim_{p\to\infty} x_i(k+1+\delta_p) \\ &= \lim_{p\to\infty} x_i(k+\delta_p)\exp\Big[b_i(k+\delta_p) - \sum_{l=1}^{n} a_{il}(k+\delta_p)x_l^{\alpha_{il}}(k+\delta_p) \\ &\quad - \sum_{l=1}^{n} c_{il}(k+\delta_p)x_i^{\alpha_{ii}}(k+\delta_p)x_l^{\alpha_{il}}(k+\delta_p) - \sum_{l=1}^{m} d_{il}(k+\delta_p)y_l^{\beta_{il}}(k+\delta_p)\Big] \\ &= p_i(k)\exp\Big[b_i(k) - \sum_{l=1}^{n} a_{il}(k)p_l^{\alpha_{il}}(k) - \sum_{l=1}^{n} c_{il}(k)p_i^{\alpha_{ii}}(k)p_l^{\alpha_{il}}(k) - \sum_{l=1}^{m} d_{il}(k)u_l^{\beta_{il}}(k)\Big], \\ u_j(k+1) &= \lim_{p\to\infty} y_j(k+1+\delta_p) \\ &= \lim_{p\to\infty} y_j(k+\delta_p)\exp\Big[-r_j(k+\delta_p) - \sum_{l=1}^{n} e_{jl}(k+\delta_p)x_l^{\delta_{jl}}(k+\delta_p) \\ &\quad - \sum_{l=1}^{m} f_{jl}(k+\delta_p)y_j^{\eta_{jj}}(k+\delta_p)y_l^{\eta_{jl}}(k+\delta_p) - \sum_{l=1}^{m} g_{jl}(k+\delta_p)y_l^{\eta_{jl}}(k+\delta_p)\Big] \\ &= u_j(k)\exp\Big[-r_j(k) - \sum_{l=1}^{n} e_{jl}(k)p_l^{\delta_{jl}}(k) - \sum_{l=1}^{m} f_{jl}(k)u_j^{\eta_{jj}}(k)u_l^{\eta_{jl}}(k) - \sum_{l=1}^{m} g_{jl}(k)u_l^{\eta_{jl}}(k)\Big], \end{aligned}$$

This prove that $p(k) = \{(p_1(k), p_2(k), \cdots, p_n(k), u_1(k), u_2(k), \cdots, u_m(k))\}$ satisfied system (1.2), and $p(k)$ is a positive almost periodic solution of system (1.2).

Now, we show that there is only one positive almost periodic solution of system (1.2). For any two positive almost periodic solutions $(p_1(k), p_2(k), \cdots, p_n(k), u_1(k), u_2(k), \cdots, u_m(k))$ and $(z_1(k), z_2(k), \cdots, z_n(k), w_1(k), w_2(k), \cdots, w_m(k))$ of system (1.2), we claim that $p_i(k) = z_i(k)$, $u_j(k) = w_j(k)(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ for all $k \in \mathbf{Z}^+$. Otherwise there must be at least one positive integer $K^* \in \mathbf{Z}^+$ such that $p_i(K^*) \neq z_i(K^*)$ or $u_j(K^*) \neq w_j(K^*)$ for a certain positive integer i or j, i.e., $\Omega_1 = |p_i(K^*) - z_i(K^*)| > 0$ or $\Omega_2 = |u_j(K^*) - w_j(K^*)| > 0$. So we can easily know that

$$\Omega_1 = |\lim_{p\to+\infty} p_i(K^*+\delta_p) - \lim_{p\to+\infty} z_i(K^*+\delta_p)| = \lim_{p\to+\infty} |p_i(K^*+\delta_p) - z_i(K^*+\delta_p)|$$

$= \lim_{k\to+\infty} |p_i(k) - z_i(k)| > 0,$
or

$$\Omega_2 = |\lim_{p\to+\infty} u_j(K^*+\delta_p) - \lim_{p\to+\infty} w_j(K^*+\delta_p)| = \lim_{p\to+\infty} |u_j(K^*+\delta_p) - w_j(K^*+\delta_p)|$$

$= \lim_{k\to+\infty} |u_j(k) - w_j(k)| > 0,$
which is a contradiction to (4.1) or (4.2). Thus $p_i(k) = z_i(k)$, $u_j(k) = w_j(k)(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ hold for $\forall k \in \mathbf{Z}^+$. Therefore, system (1.2) admits a unique almost periodic solution which is globally attractive. This completes the proof of Theorem 4.2. □

Remark 4.3 If $m = n = 1$, the conditions of Theorem 4.2 can be simplified. Therefore, we have the following results.

Corollary 4.4 Let $m = n = 1$, assume that (H1)-(H4) and

(H5) $$\begin{aligned} \lambda_1 &= \max\Big\{\Big|1-\alpha_{11}\Big(a_{11}^u(M_1)^{\alpha_{11}}+2c_{11}^u(M_1)^{2\alpha_{11}}\Big)\Big|, \Big|1-\alpha_{11}\Big(a_{11}^l(m_1)^{\alpha_{11}}+2c_{11}^l(m_1)^{2\alpha_{11}}\Big)\Big|\Big\} \\ &\quad +\beta_{11}d_{11}^u(N_1)^{\beta_{11}} < 1, \\ \gamma_1 &= \max\Big\{\Big|1-\eta_{11}\Big(g_{11}^u(N_1)^{\eta_{11}}+2f_{11}^u(N_1)^{2\eta_{11}}\Big)\Big|, \Big|1-\eta_{11}\Big(g_{11}^l(n_1)^{\eta_{11}}+2f_{11}^u(n_1)^{2\eta_{11}}\Big)\Big|\Big\} \end{aligned}$$

$$+e_{11}^{u}(M_1)^{\delta_{11}} < 1,$$

hold. Then system (1.2) admits a unique almost periodic solution which is globally attractive.

5 Numerical Simulations

In this section, we give the following examples to check the feasibility of our results.
Example 5.1 Consider the discrete Gilpin-Ayala type competition-predator system:

$$\begin{cases} x(k+1) = x(k)\exp\Big\{1.2 - 0.02\sin(\sqrt{2}k) - (0.25 + 0.01\sin(\sqrt{3}k))x^{\frac{1}{2}}(k) \\ \qquad -(0.025 + 0.002\cos(\sqrt{5}k))x^2(k) - (0.02 + 0.001\cos(\sqrt{2}k))y(k)\Big\}, \\ y(k+1) = y(k)\exp\Big\{ - 0.1 + 0.025\sin(\sqrt{3}k) + (1.2 + 0.003\cos(\sqrt{2}k))x(k) \\ \qquad -(0.18 + 0.015\sin(\sqrt{2}k))y^2(k) - (0.025 + 0.002\cos(\sqrt{5}k))y(k)\Big\}. \end{cases} \tag{5.1}$$

A computation shows that

$$\lambda_1 \approx 0.0632 < 1, \ \gamma_1 \approx 0.0962 < 1.$$

Hence, there exists a unique globally attractive almost periodic solution of system (5.1). Our numerical simulations support our results (see Figs. 1 and 2).

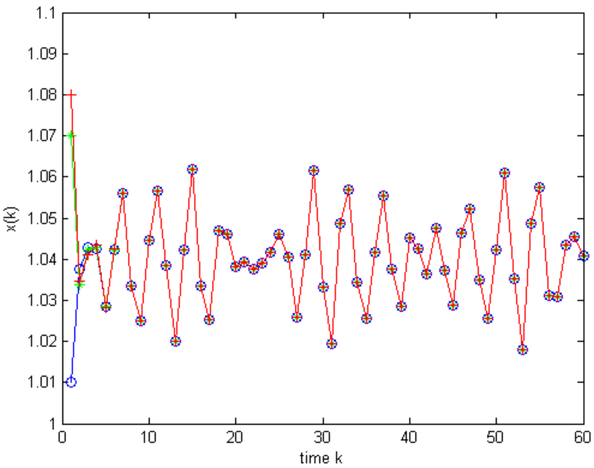

Fig. 1. Dynamic behavior of the first component $x(k)$ of the solution $(x(k), y(k))$ to system (5.1) with the initial conditions (1.01,1.07), (1.07,1.15) and (1.08,1.23) for $k \in [1, 60]$, respectively.

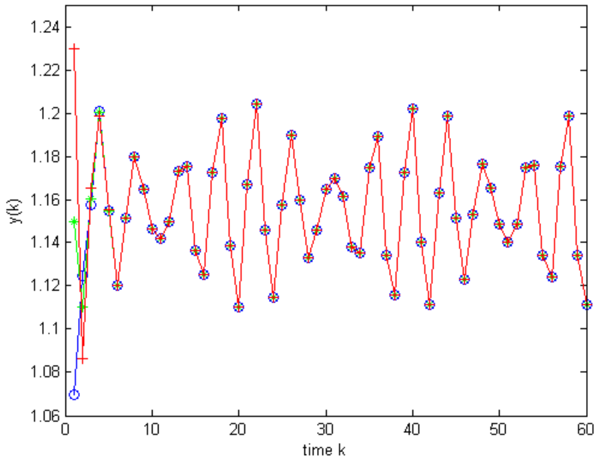

Fig. 2. Dynamic behavior of the second component $y(k)$ of the solution $(x(k), y(k))$ to system (5.1) with the initial conditions (1.01,1.07), (1.07,1.15) and (1.08,1.23) for $k \in [1, 60]$, respectively.

Example 5.2 Consider the discrete Lotka-Volterra type competition-predator system:

$$\begin{cases} x(k+1) = x(k) \exp\Big\{1.3 - 0.025\sin(\sqrt{3}k) - (0.221 + 0.013\sin(\sqrt{2}k))x(k) \\ \qquad\qquad -(0.02 + 0.001\cos(\sqrt{5}k))y(k)\Big\}, \\ y(k+1) = y(k) \exp\Big\{ -0.15 + 0.02\sin(\sqrt{2}k) + (1.25 + 0.003\cos(\sqrt{3}k))x(k) \\ \qquad\qquad -(0.186 + 0.012\sin(\sqrt{5}k))y(k)\Big\}. \end{cases} \quad (5.2)$$

A computation shows that

$$\lambda_1 \approx 0.0026 < 1, \ \gamma_1 \approx 0.0128 < 1.$$

Hence, there exists a unique globally attractive almost periodic solution of system (5.2). Our numerical simulations support our results (see Figs. 3 and 4).

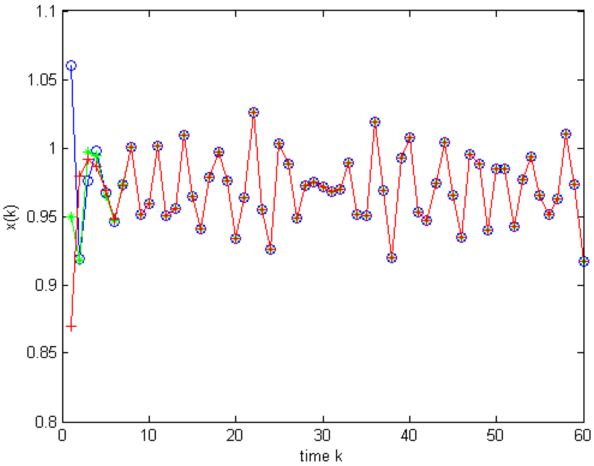

Fig. 3. Dynamic behavior of the first component $x(k)$ of the solution $(x(k), y(k))$ to system (5.2) with the initial conditions (1.06,5.7), (0.95,7.1) and (0.87,4.3) for $k \in [1, 60]$, respectively.

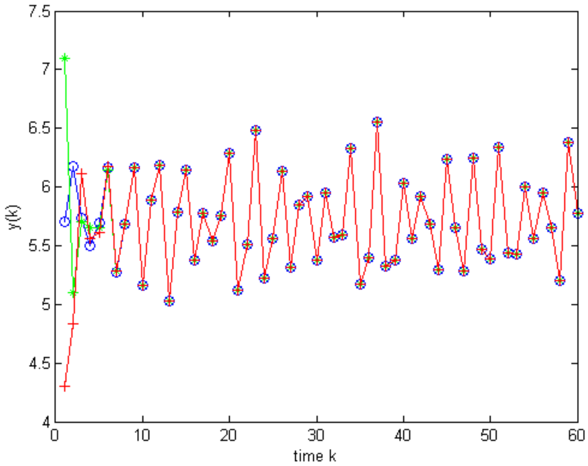

Fig. 4. Dynamic behavior of the second component $y(k)$ of the solution $(x(k), y(k))$ to system (5.2) with the initial conditions (1.06,5.7), (0.95,7.1) and (0.87,4.3) for $k \in [1, 60]$, respectively.

6 Concluding Remarks

In this paper, a discrete multispecies Gilpin-Ayala type competition-predator system is considered. By applying the difference inequality, some sufficient conditions are established to ensure the permanence of system (1.2). Then, we show that system (1.2) is globally attractive under some appropriate conditions. Assuming that the coefficients in the system are almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive by using the properties of almost periodic sequence. By comparative analysis, we find that when the coefficients in system (1.2) are almost periodic, the existence of a unique almost periodic solution of system (1.2) is determined by the global attractivity of system (1.2), which implies that there is no additional condition to add.

Furthermore, for the almost periodic discrete multispecies Gilpin-Ayala type competition-predator system (1.2) with time delays or feedback controls, we would like to mention here the question of whether the existence of a unique almost periodic solution is determined by the global attractivity of the system or not. It is, in fact, a very challenging problem, and we leave it for our future work.

Acknowledgment

The authors are grateful to the anonymous referees for their excellent suggestions, which greatly improved the presentation of the paper. Also, the authors declare that there is no conflict of interests regarding the publication of this paper, and there are no financial interest conflicts between the authors and the commercial identity. This work was partially supported by National Natural Science Foundation of China(no.11301415), Natural Science Basic Research Plan in Shaanxi Province of China(no.2013JQ1020) and Scientific Research Program Funded by Shaanxi Provincial Education Department of China(no.2013JK1098).

Competing Interests

The authors declare that no competing interests exist.

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