

Research Article

Charged-Particle Multiplicity Moments as Described by Shifted Gompertz Distribution in e^+e^- , $p\bar{p}$, and pp Collisions at High Energies

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In continuation of our earlier work, in which we analysed the charged particle multiplicities in leptonic and hadronic interactions at different center-of-mass energies in full phase space as well as in restricted phase space using the shifted Gompertz distribution, a detailed analysis of the normalized moments and normalized factorial moments is reported here. A two-component model in which a probability distribution function is obtained from the superposition of two shifted Gompertz distributions, as introduced in our earlier work, has also been used for the analysis. This is the first analysis of the moments with the shifted Gompertz distribution. Analysis has also been performed to predict the moments of multiplicity distribution for the e^+e^- collisions at $\sqrt{s} = 500$ GeV at a future collider.

1. Introduction

In one of our recent papers, we introduced a statistical distribution, the shifted Gompertz distribution to investigate the multiplicity distributions of charged particles produced in e^+e^- collisions at the LEP, $p\bar{p}$ interactions at the SPS and pp collisions at the LHC at different center of mass energies in full phase space as well as in restricted phase space [1]. A distribution of the larger of two independent random variables, the shifted Gompertz distribution was introduced by Bemmaor [2] as a model of adoption of innovations. One of the parameters has an exponential distribution and the other has a Gumbel distribution, also known as log-Weibull distribution. The nonnegative fit parameters define the scale and shape of the distribution. Subsequently, the shifted Gompertz distribution has been widely studied in various contexts [3–5]. In our earlier work [1] by studying the charged particle multiplicities, we showed that this distribution can be successfully used to study the statistical phenomena in high energy e^+e^- , $p\bar{p}$, and pp collisions at the LEP, SPS, and LHC colliders, respectively.

A multiplicity distribution is represented by the probabilities of n -particle events as well as by its moments or its

generating function. The aim of the present work is to extend the analysis by calculating the higher moments of a multiplicity distribution. Because the moments are calculated as derivatives of the generating function, the moment analysis is a powerful tool which helps to unfold the characteristics of multiplicity distribution. The multiparticle correlations can be studied through the normalized moments and normalized factorial moments of the distribution. The dependence of moments on energy can also reveal the KNO (Koba, Nielsen, and Olesen) scaling [6–8] conservation or violation. Several analyses of moments have been done at different energies, using different probability distribution functions and different types of particles [9–12]. The higher moments also can identify the correlations amongst produced particles.

In Section 2, formulae for the probability distribution function (PDF) of the shifted Gompertz distribution, normalized moments, and the normalized factorial moments used for the analysis are given. A two-component model has been used and modification of distributions carried out, in terms of these two components; one from soft events and another from semi-hard events. Superposition of distributions from these two components, using appropriate weights is done to

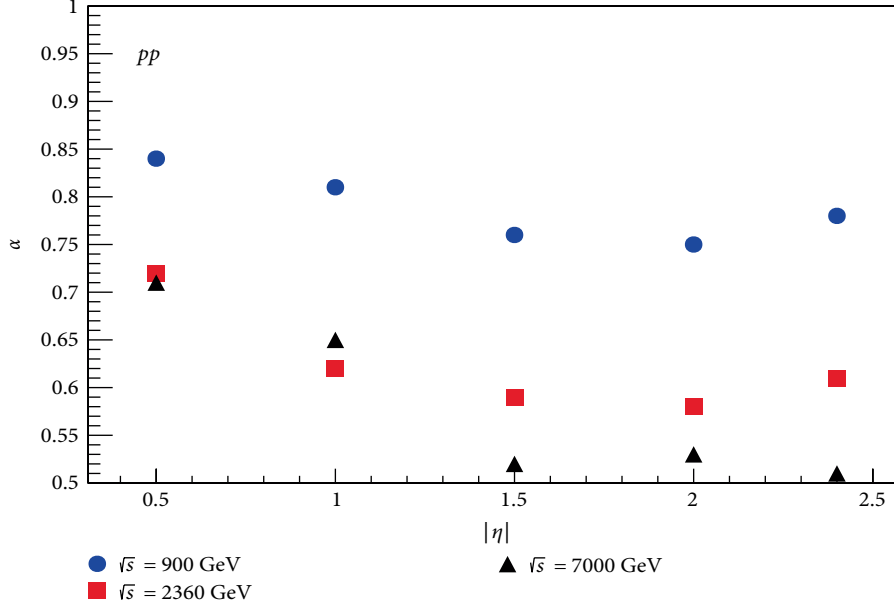


FIGURE 1: Dependence of α on c.m. energy and pseudorapidity for pp interaction.

build the full multiplicity distribution. When multiplicity distribution is fitted with the weighted superposition of two shifted Gompertz distributions, we find that the agreement between data and the model improves considerably. The details of these fits are published in [1]. The distributions have been fitted both in full phase space as well as in restricted rapidity windows for $p\bar{p}$ and pp data and in five rapidity windows and in full phase space only for e^+e^- , in terms of soft and semi-hard components.

Section 3 presents the evaluations of moments from experimental data, the fitted shifted Gompertz distributions and the fitted modified shifted Gompertz distributions. Section 4 details the method of estimating uncertainties on the moments. Discussion and conclusion are presented in Section 5.

2. Shifted Gompertz Distribution and Moments

The particle production dynamics can be understood by analysing the charged particle multiplicity distribution as its measurements can provide relevant constraints for particle-production models. Charged particle multiplicity is defined as the average number of charged particles n , produced in a collision at a given energy in the center of mass system.

In addition, the analysis of moments of the distribution is often used to study the patterns and correlations in the multi-particle final state of high-energy collisions in the presence of statistical fluctuations. The fractal structures present in the multiplicity distributions have often been studied to search for the embedded constraints on the underlying particle production mechanism [13, 14]. The observation of fractal structures is of great interest because it imposes strong constraints on the underlying particle-production mechanism. We define different kinds of moments as follows.

Let X be any nonnegative random variable having the shifted Gompertz distribution with parameters b and β , where $b > 0$ is a scale parameter and $\beta > 0$ is a shape parameter. The probability distribution function (PDF) of X is given by

$$P_X(x; b, \beta) = be^{-(bx + \beta e^{-bx})} (1 + \beta(1 - e^{-bx})), \quad \text{where } x > 0. \quad (1)$$

The raw moments (c_n) and factorial moments (f_n) are defined as:

$$c_n = E[X^n] \quad \text{and} \quad f_n = E[(X)(X-1)(X-2)\dots(X-(n-1))], \quad (2)$$

whereas, the normalized moments (C_n) and normalized factorial moments (F_n) are defined as following:

$$C_n = \frac{E[X^n]}{(E[X])^n} \quad \text{and} \quad F_n = \frac{E[(X)(X-1)(X-2)\dots(X-(n-1))]}{(E[X])^n}, \quad (3)$$

n as a natural number ranging from 1 to ∞ . The mean value ($E[X]$) of shifted Gompertz distribution is given by

$$E[X] = \frac{1}{b} \left(\gamma + \ln \beta + \frac{1 - e^{-\beta}}{\beta} + \Gamma[0, \beta] \right) \quad (4)$$

and the c_2 moment is given by

$$c_2 = E[X^2] = \frac{2}{b^2 \beta} (\gamma + \Gamma[0, \beta] + \beta^2 {}_3F_3[\{1, 1, 1\}, \{2, 2, 2\}, -\beta] + \ln \beta). \quad (5)$$

The higher order raw moments (c_n) can be found by the moment generating function of the shifted Gompertz distribution $[[b, \beta], t]$

$$e^{-\beta} - \left(1 + \frac{t}{b\beta}\right) \beta^{t/b} \left(\Gamma\left[1 - \frac{t}{b}\right] - \Gamma\left[1 - \frac{t}{b}, \beta\right] \right). \quad (6)$$

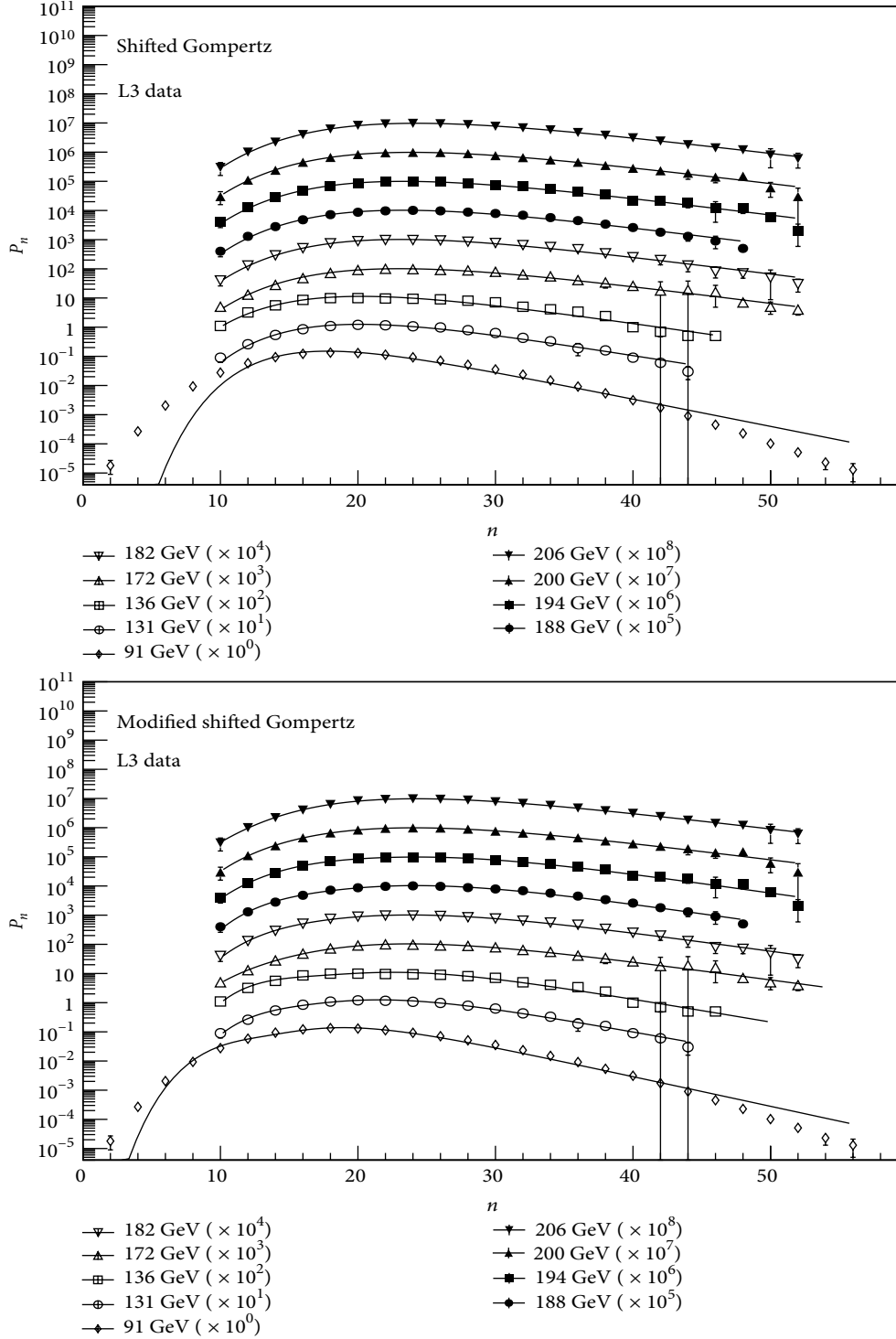


FIGURE 2: Charged multiplicity distributions from L3 experiment. Solid lines represent the shifted Gompertz distribution and the modified shifted Gompertz distributions.

Also f_2 moment is given by

$$f_2 = E[(X)(X-1)] = \frac{2}{b^2\beta}(\gamma + \Gamma[0, \beta] + \beta^2 {}_3F_3[\{1, 1, 1\}, \{2, 2, 2\}, -\beta] + \ln\beta) - \frac{(1 - e^{-\beta} + \beta(\gamma + \Gamma[0, \beta] + \ln\beta))}{b\beta}. \quad (7)$$

The higher factorial moments (f_n) can be found by the generating function of shifted Gompertz distribution $[[b, \beta], t]$

$$e^{-\beta} - \beta^{\ln t/b} \left(\Gamma\left[1 - \frac{\ln t}{b}\right] - \Gamma\left[1 - \frac{\ln t}{b}, \beta\right] \right) \left(1 + \frac{\ln t}{b\beta}\right), \quad (8)$$

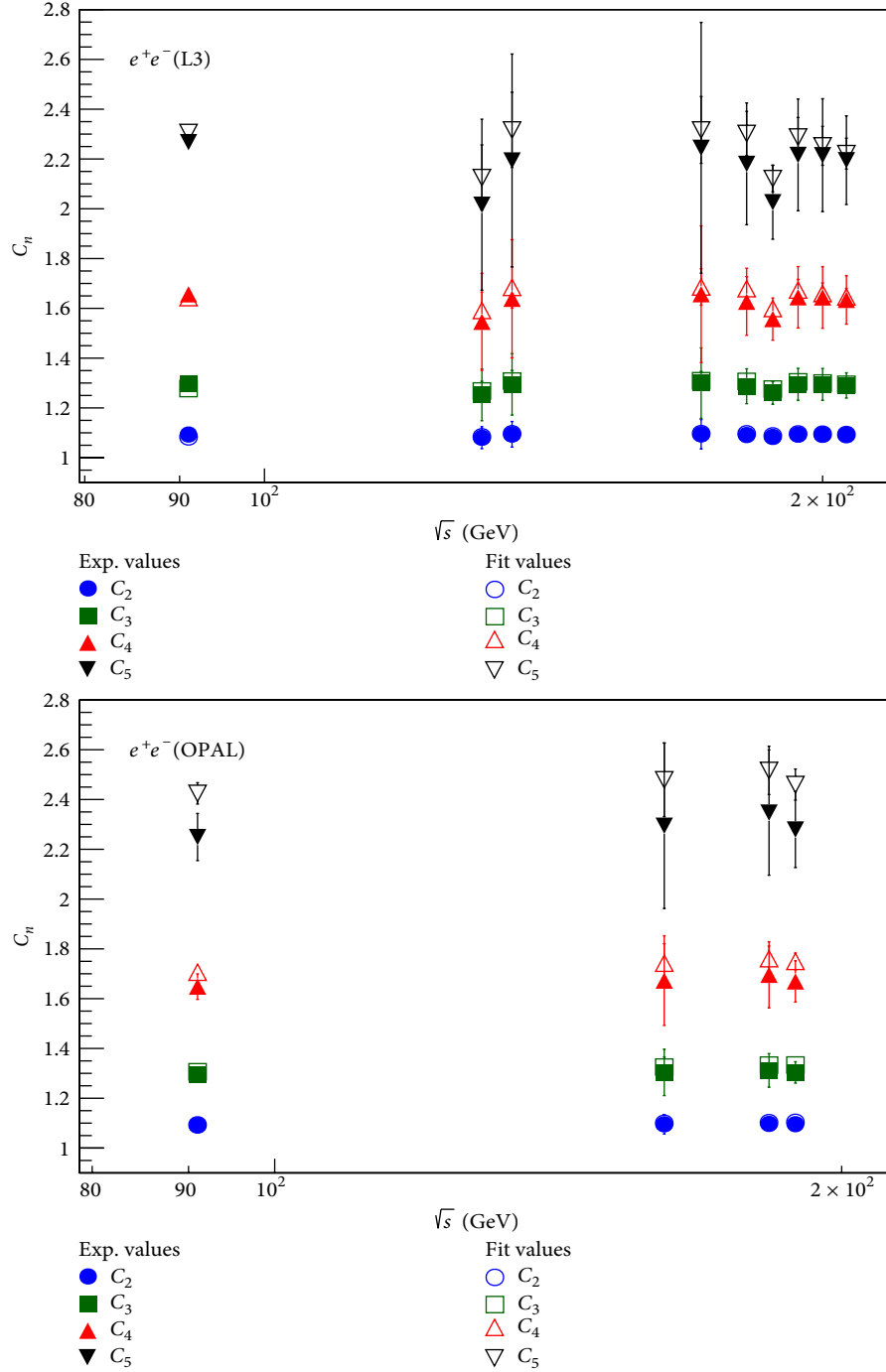


FIGURE 3: Normalized moments of shifted Gompertz distribution for the e^+e^- collisions recorded by the L3 and OPAL experiments at different energies.

where

- (i) $\gamma \approx 0.5772156$ stands for the Euler constant (also referred to as Euler-Mascheroni constant).
- (ii) $\Gamma[s]$ the Euler Gamma function and $\Gamma[s, x]$ the incomplete Gamma function defined below;

$$\begin{aligned} \Gamma[s] &= \int_0^{\infty} t^{s-1} e^{-t} dt \\ \Gamma[s, x] &= \int_x^{\infty} t^{s-1} e^{-t} dt, \end{aligned} \quad (9)$$

(iii) ${}_3F_3$ is a generalized hypergeometric function.

$${}_3F_3[\{1, 1, 1\}, \{2, 2, 2\}, -\beta] = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \beta^{k+1}}{k! k^2}. \quad (10)$$

2.1. Modified Shifted Gompertz Distribution. It is well established that at high energies, charged particle multiplicity distribution in full phase space becomes broader than a Poisson distribution. The most widely adopted, negative binomial distribution [15] to describe the multiplicity

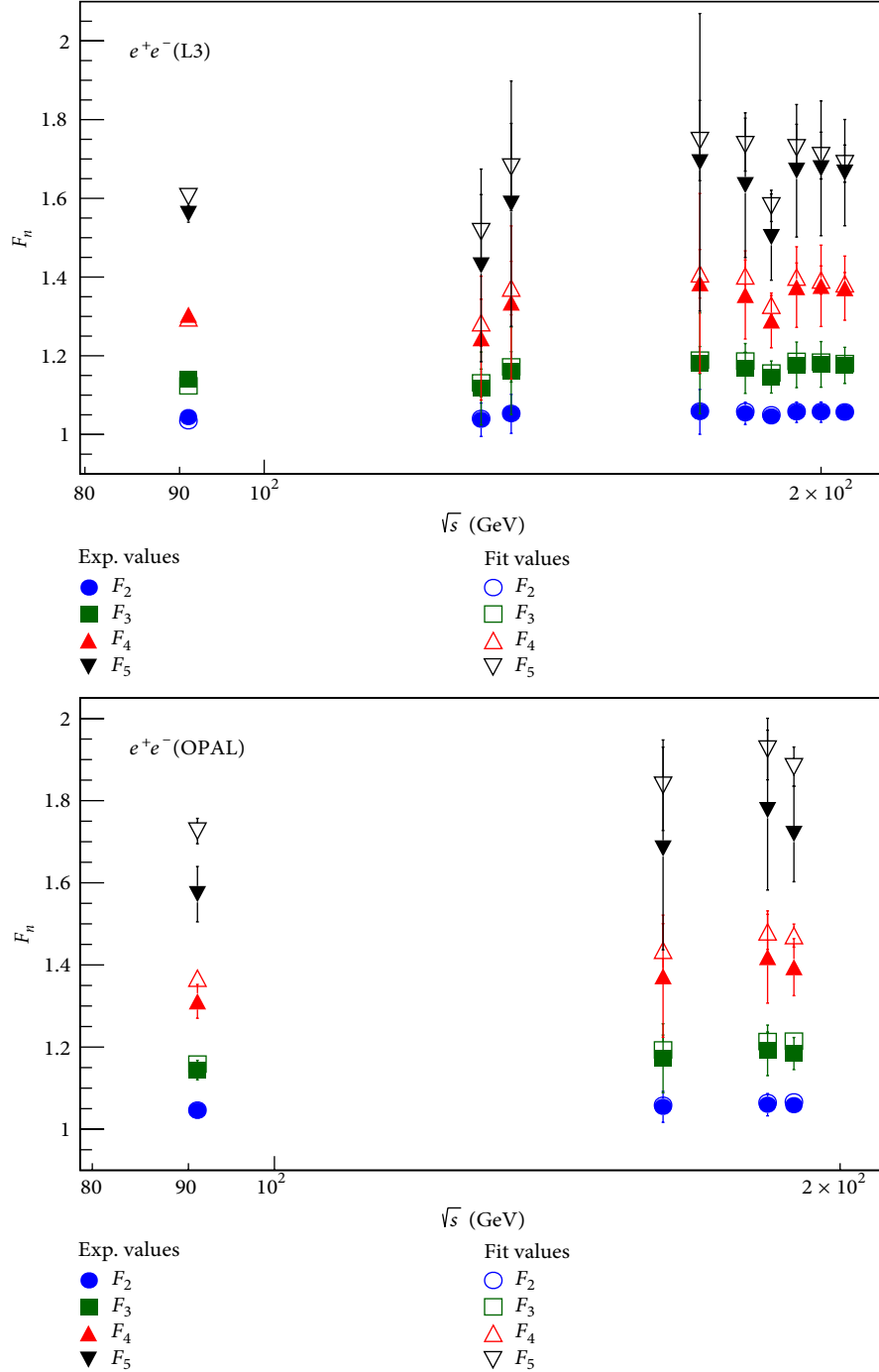


FIGURE 4: Normalized factorial moments of shifted Gompertz distribution for the e^+e^- collisions recorded by the L3 and OPAL experiments at different energies.

spectra, fails to explain the experimental data. As a corrective measure to explain the failure, a two-component approach, was introduced by Giovannini et al. [15]. The details are included in our earlier publication [1] on the shifted Gompertz distribution.

To better explain the data at high energies, a superposition of two shifted Gompertz distributions, which are interpreted as soft and hard components, is used. The multiplicity

distribution is produced by adding weighted superposition of multiplicity in soft events and multiplicity distribution in semi-hard events. This approach combines merely two classes of events and not two different particle-production mechanisms. Therefore, no interference terms need to be introduced. The final distribution is the superposition of the two independent distributions. We call it “modified shifted Gompertz distribution”.

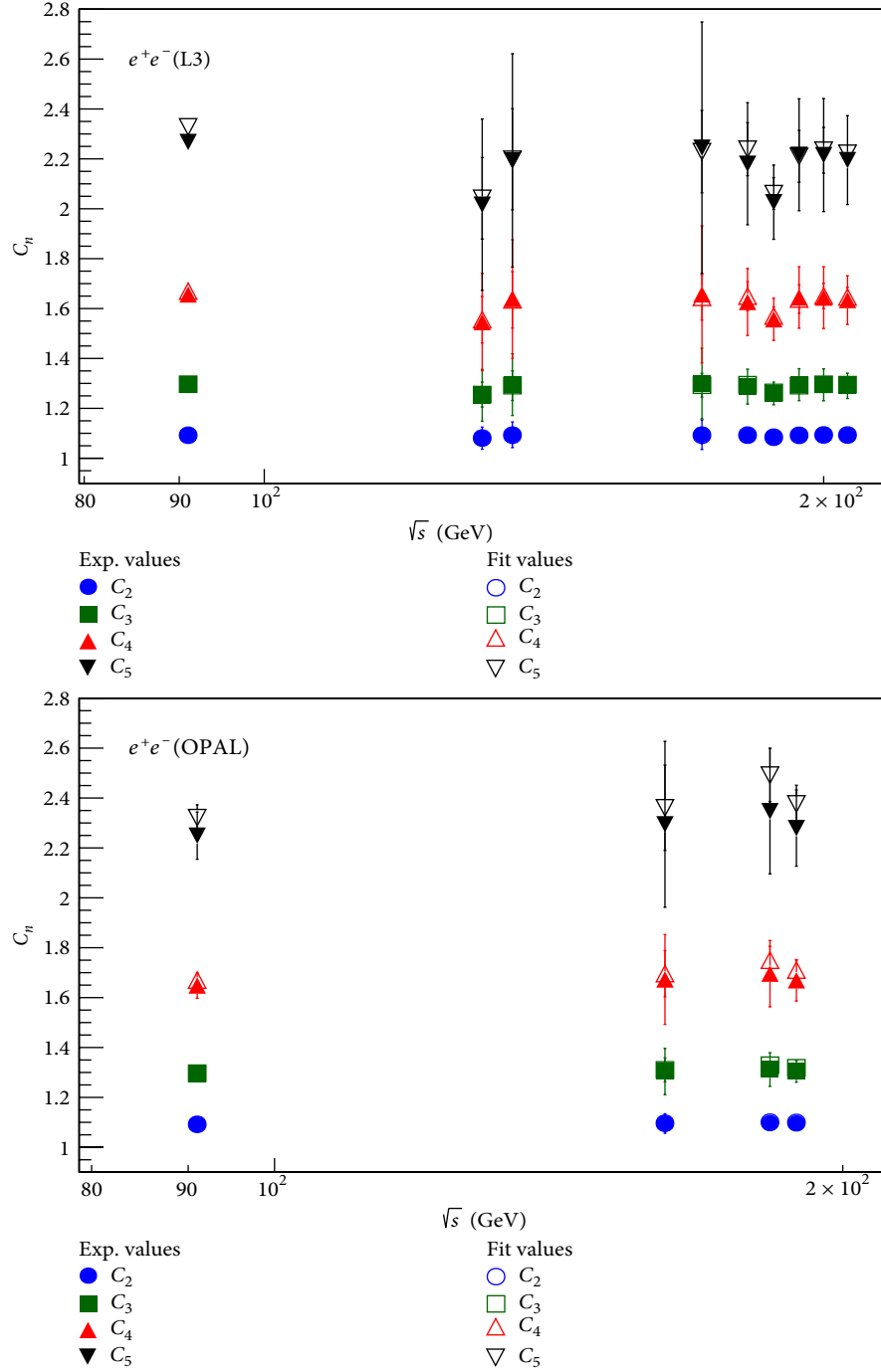


FIGURE 5: Normalized moments of modified shifted Gompertz distribution for the e^+e^- collisions recorded by the L3 and OPAL experiments at different energies.

$$P(n) = \alpha P_{\text{soft}}^{\text{shGomp}}(n) + (1 - \alpha) P_{\text{semi-hard}}^{\text{shGomp}}(n). \quad (11)$$

Adopting this approach for the multiplicity distributions in e^+e^- , pp , and $p\bar{p}$ collisions at high energies, the data at different energies are fitted with the distribution which involves five parameters as given below;

$$P_n(\alpha : b_1, \beta_1; b_2, \beta_2) = \alpha P_n(\text{soft}) + (1 - \alpha) P_n(\text{semi-hard}), \quad (12)$$

where α is the fraction of soft events, (b_1, β_1) and (b_2, β_2) are respectively the scale and shape parameters of the two distributions.

Values of α evaluated for different interactions, were published in the Tables in our previous paper [1]. In Figure 2,

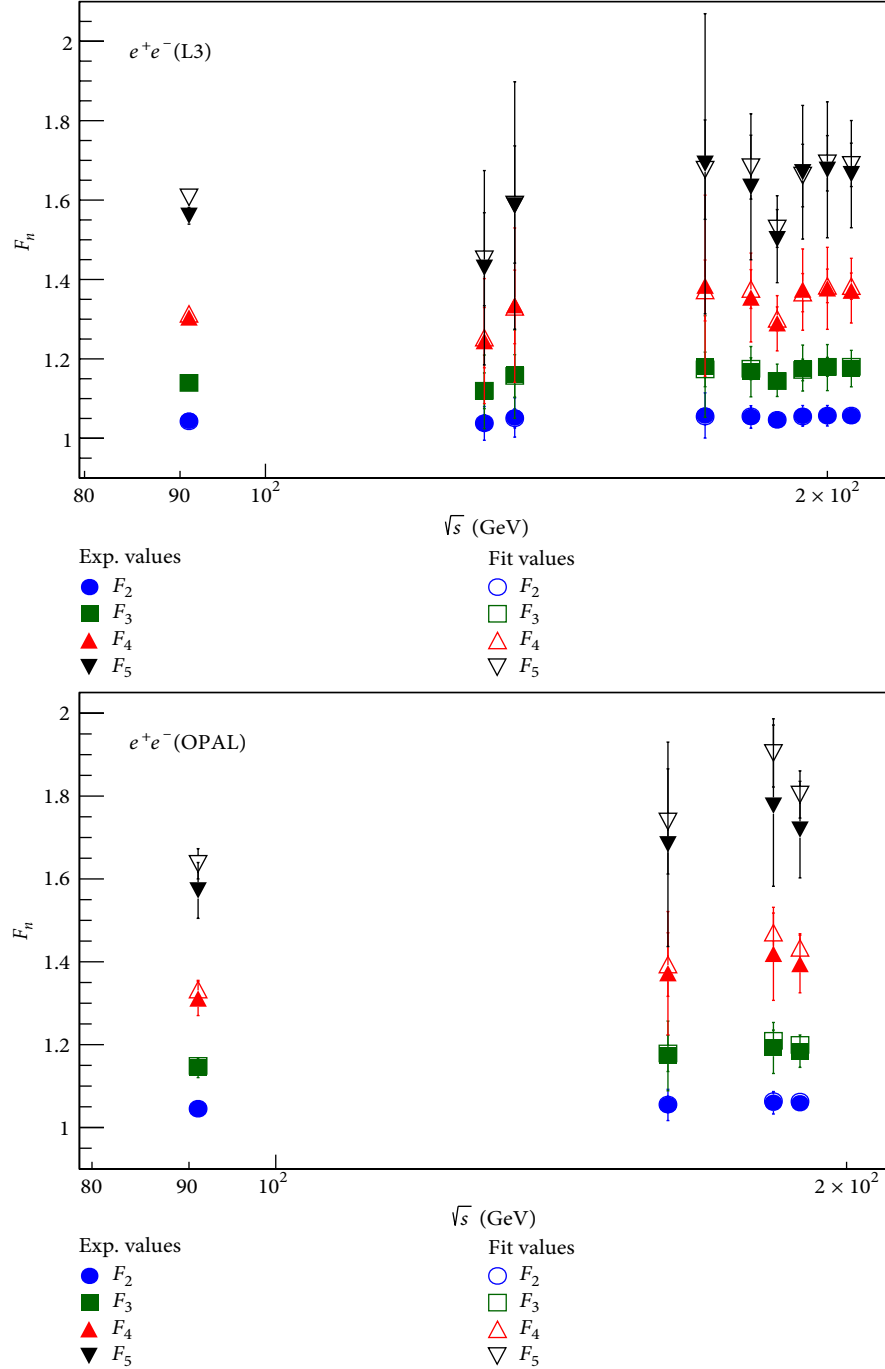


FIGURE 6: Normalized factorial moments of modified shifted Gompertz distribution for the e^+e^- collisions recorded by the L3 and OPAL experiments at different energies.

we show the variation of α as a function of collision energy and pseudorapidity for pp interactions at LHC energies. The values show that the alpha decreases with collision energy as well as with increasing pseudo-rapidity window.

3. Analysis and Results

Calculations of the normalized moments and the normalized factorial moments are presented by using the data from different experiments and following three collision types;

- (i) e^+e^- annihilations at different collision energies, from 91 GeV up to the highest energy of 206 GeV at LEP2, from two experiments L3 [16] and OPAL [17–20] are analysed.
- (ii) pp collisions at LHC energies from 900 GeV, 2360 GeV, and 7000 GeV [21] are analysed for five intervals of increasing extent in pseudorapidity $|\eta| < 0.5$ up to $|\eta| < 2.4$.
- (iii) $p\bar{p}$ collisions at energies from 200 GeV, 540 GeV, and 900 GeV [7, 8] are analysed in full phase space as

TABLE 1: Moments of experimental, shifted Gompertz and modified shifted Gompertz distributions for e^+e^- collisions.

Energy (GeV) (e^+e^-)	$\langle n \rangle$	Normalized moments (experiment)					Normalized factorial moments (experiment)				
		C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5		
L3	91	20.464±0.076	1.093±0.004	1.296±0.008	1.656±0.016	2.268±0.030	1.044±0.003	1.141±0.007	1.304±0.013	1.560±0.021	
	131	23.301±1.002	1.080±0.044	1.252±0.104	1.546±0.195	2.016±0.343	1.038±0.043	1.117±0.093	1.245±0.157	1.430±0.245	
	136	24.012±1.116	1.094±0.051	1.295±0.124	1.638±0.237	2.194±0.428	1.052±0.050	1.162±0.111	1.335±0.195	1.586±0.312	
	172	27.031±1.821	1.095±0.059	1.299±0.142	1.657±0.275	2.245±0.504	1.058±0.057	1.181±0.128	1.384±0.228	1.692±0.378	
	182	26.795±0.863	1.091±0.030	1.287±0.070	1.626±0.135	2.180±0.245	1.054±0.028	1.168±0.063	1.355±0.112	1.633±0.184	
	188	26.653±0.609	1.084±0.020	1.260±0.045	1.557±0.085	2.026±0.149	1.046±0.019	1.141±0.041	1.290±0.070	1.502±0.110	
OPAL	194	27.178±0.847	1.093±0.027	1.295±0.064	1.644±0.123	2.216±0.224	1.057±0.026	1.177±0.058	1.375±0.102	1.670±0.168	
	200	27.575±0.852	1.093±0.027	1.294±0.064	1.644±0.124	2.215±0.227	1.057±0.026	1.178±0.058	1.378±0.103	1.676±0.171	
	206	27.917±0.720	1.092±0.021	1.290±0.051	1.634±0.098	2.195±0.178	1.056±0.021	1.176±0.046	1.372±0.082	1.665±0.135	
	91	21.394±0.219	1.092±0.011	1.293±0.027	1.648±0.051	2.249±0.095	1.045±0.011	1.144±0.024	1.311±0.041	1.572±0.067	
	161	24.463±0.859	1.095±0.039	1.304±0.093	1.673±0.180	2.295±0.333	1.054±0.038	1.173±0.084	1.373±0.149	1.684±0.247	
	183	26.856±0.754	1.097±0.028	1.312±0.068	1.696±0.133	2.348±0.252	1.060±0.027	1.192±0.062	1.419±0.112	1.777±0.195	
L3	189	26.886±0.505	1.096±0.018	1.304±0.043	1.669±0.083	2.280±0.153	1.059±0.018	1.184±0.039	1.395±0.070	1.719±0.117	
	91	20.467±0.029	1.084±0.002	1.278±0.004	1.643±0.008	2.307±0.015	1.035±0.002	1.124±0.004	1.296±0.006	1.604±0.010	
	131	23.101±0.325	1.084±0.017	1.268±0.039	1.591±0.074	2.126±0.131	1.041±0.016	1.131±0.035	1.284±0.060	1.515±0.094	
	136	23.530±0.348	1.096±0.018	1.308±0.043	1.684±0.083	2.317±0.152	1.054±0.017	1.172±0.039	1.372±0.068	1.680±0.111	
	172	26.706±0.359	1.097±0.016	1.309±0.038	1.686±0.073	2.317±0.134	1.059±0.015	1.189±0.035	1.408±0.062	1.747±0.102	
	182	26.779±0.259	1.096±0.010	1.307±0.025	1.679±0.048	2.303±0.089	1.059±0.010	1.187±0.023	1.403±0.040	1.737±0.067	
OPAL	188	26.624±0.172	1.088±0.007	1.276±0.016	1.599±0.030	2.121±0.053	1.050±0.006	1.156±0.014	1.328±0.025	1.581±0.040	
	194	27.007±0.232	1.095±0.009	1.305±0.022	1.674±0.043	2.288±0.078	1.058±0.009	1.186±0.020	1.400±0.036	1.729±0.060	
	200	27.599±0.251	1.095±0.009	1.300±0.022	1.660±0.043	2.253±0.078	1.058±0.009	1.184±0.020	1.392±0.036	1.709±0.060	
	206	27.940±0.209	1.093±0.007	1.295±0.018	1.646±0.034	2.222±0.062	1.057±0.007	1.180±0.016	1.383±0.029	1.688±0.047	
	91	21.547±0.086	1.093±0.005	1.306±0.012	1.706±0.023	2.425±0.043	1.046±0.005	1.159±0.010	1.368±0.018	1.726±0.031	
	161	24.322±0.325	1.099±0.017	1.325±0.040	1.742±0.078	2.479±0.148	1.058±0.016	1.193±0.036	1.436±0.065	1.838±0.110	
OPAL	183	26.980±0.244	1.102±0.011	1.333±0.026	1.761±0.051	2.517±0.097	1.065±0.010	1.213±0.023	1.481±0.043	1.926±0.075	
	189	27.176±0.164	1.103±0.007	1.334±0.017	1.750±0.033	2.466±0.062	1.066±0.007	1.214±0.016	1.471±0.028	1.883±0.048	

TABLE 1: Continued.

Energy (GeV) (e^+e^-)	$\langle n \rangle$	Normalized moments (modified shifted Gompertz)					Normalized factorial moments (modified shifted Gompertz)				
		C_2	C_3	C_4	C_5		F_2	F_3	F_4	F_5	
L3	91	1.092 ± 0.002	1.297 ± 0.005	1.670 ± 0.009	2.329 ± 0.016		1.043 ± 0.002	1.139 ± 0.004	1.313 ± 0.007	1.607 ± 0.011	
	131	1.081 ± 0.022	1.255 ± 0.050	1.555 ± 0.093	2.042 ± 0.164		1.038 ± 0.021	1.120 ± 0.045	1.253 ± 0.076	1.451 ± 0.117	
	136	1.092 ± 0.025	1.291 ± 0.059	1.635 ± 0.113	2.198 ± 0.203		1.050 ± 0.024	1.157 ± 0.054	1.331 ± 0.093	1.589 ± 0.148	
	172	1.092 ± 0.020	1.293 ± 0.048	1.645 ± 0.091	2.229 ± 0.165		1.055 ± 0.019	1.174 ± 0.044	1.372 ± 0.077	1.677 ± 0.125	
	182	1.093 ± 0.013	1.295 ± 0.030	1.650 ± 0.058	2.239 ± 0.106		1.055 ± 0.012	1.175 ± 0.028	1.376 ± 0.049	1.683 ± 0.081	
	188	1.084 ± 0.008	1.264 ± 0.020	1.570 ± 0.036	2.061 ± 0.064		1.047 ± 0.008	1.144 ± 0.018	1.301 ± 0.030	1.528 ± 0.048	
	194	1.092 ± 0.013	1.291 ± 0.030	1.638 ± 0.057	2.210 ± 0.104		1.055 ± 0.012	1.172 ± 0.027	1.367 ± 0.048	1.662 ± 0.079	
	200	1.093 ± 0.011	1.296 ± 0.026	1.651 ± 0.050	2.235 ± 0.092		1.057 ± 0.011	1.180 ± 0.024	1.384 ± 0.042	1.693 ± 0.070	
	206	1.093 ± 0.009	1.295 ± 0.021	1.646 ± 0.040	2.223 ± 0.072		1.057 ± 0.008	1.180 ± 0.019	1.383 ± 0.033	1.689 ± 0.055	
	91	1.092 ± 0.006	1.297 ± 0.015	1.670 ± 0.028	2.322 ± 0.051		1.045 ± 0.006	1.149 ± 0.013	1.333 ± 0.022	1.637 ± 0.037	
OPAL	161	1.096 ± 0.020	1.310 ± 0.048	1.696 ± 0.093	2.361 ± 0.171		1.055 ± 0.020	1.179 ± 0.044	1.393 ± 0.077	1.739 ± 0.127	
	183	1.101 ± 0.012	1.329 ± 0.028	1.750 ± 0.056	2.492 ± 0.107		1.064 ± 0.011	1.209 ± 0.026	1.470 ± 0.047	1.904 ± 0.082	
	189	1.100 ± 0.009	1.319 ± 0.021	1.710 ± 0.040	2.377 ± 0.075		1.063 ± 0.009	1.199 ± 0.019	1.434 ± 0.034	1.804 ± 0.057	

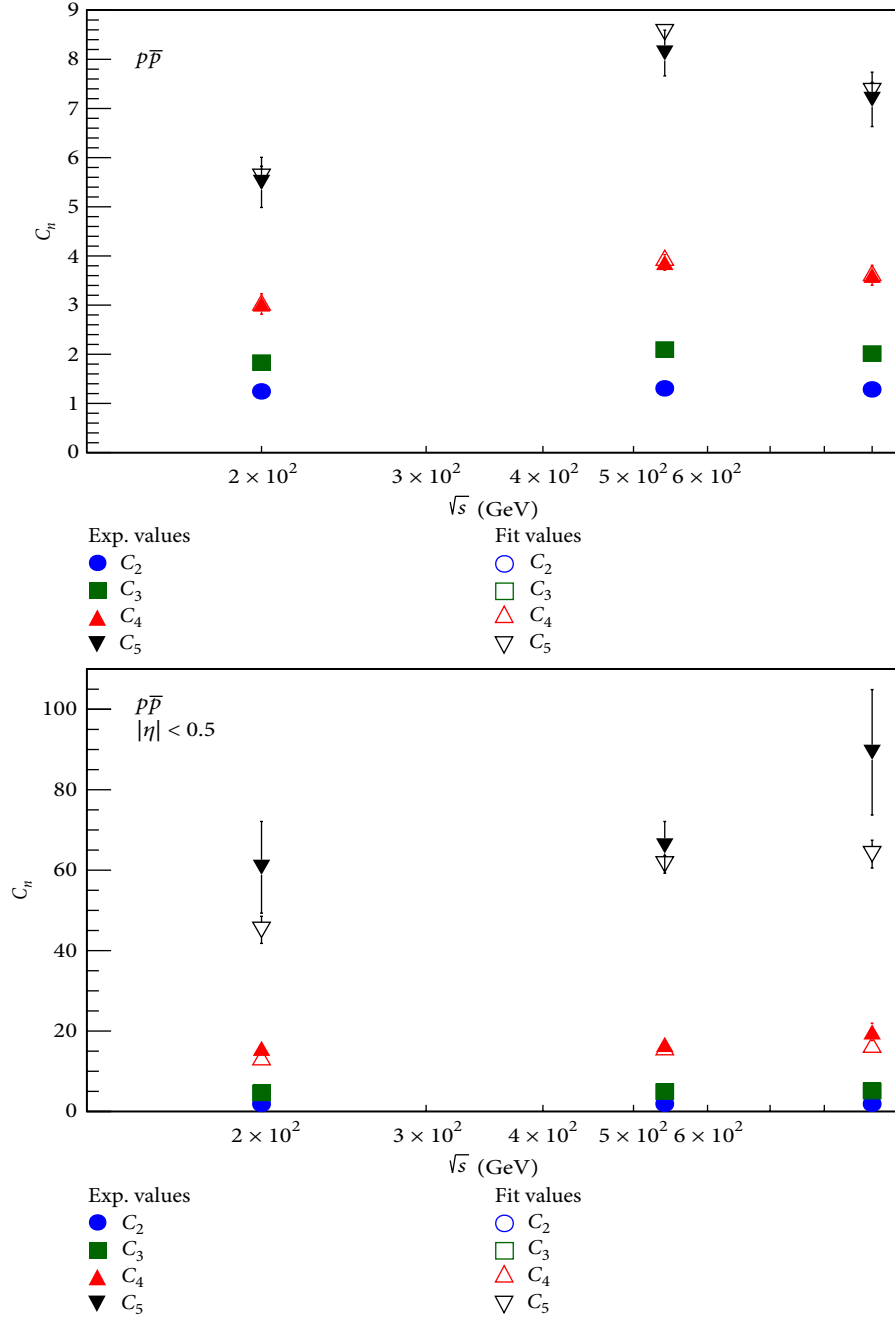


FIGURE 7: Normalized moments C_n of modified shifted Gompertz distribution for the $p\bar{p}$ collisions recorded by the UA5 experiment at different energies, in full phase space and pseudo-rapidity bin, $|\eta| < 0.5$.

well as in pseudorapidity intervals from $|\eta| < 0.5$ up to $|\eta| < 5.0$, where η is defined as $-\ln[\tan(\theta/2)]$, and θ is the polar angle of the particle with respect to the counter-clockwise beam direction.

The PDF defined by Equation (1) is used to fit the experimental data on charged particle multiplicity distributions, for the shifted Gompertz function and the modified (two-component) function. Results from these fits to the above mentioned data were published in our earlier work [1]. As an example,

results for L3 data are shown in Figure 1. To avoid repetition, the details of other Figures are not given here. It was shown that the data are very well explained by the modified shifted Gompertz distribution and the χ^2 values for the fits in almost all cases reduce substantially. In the present analysis we calculate the normalized moments and the normalized factorial moments defined in Equations (2)–(8).

Figures 3–6 show the normalized moments (C_q) and the normalized factorial moments (F_q) calculated from the data of two experiments at LEP, at different energies for e^+e^-

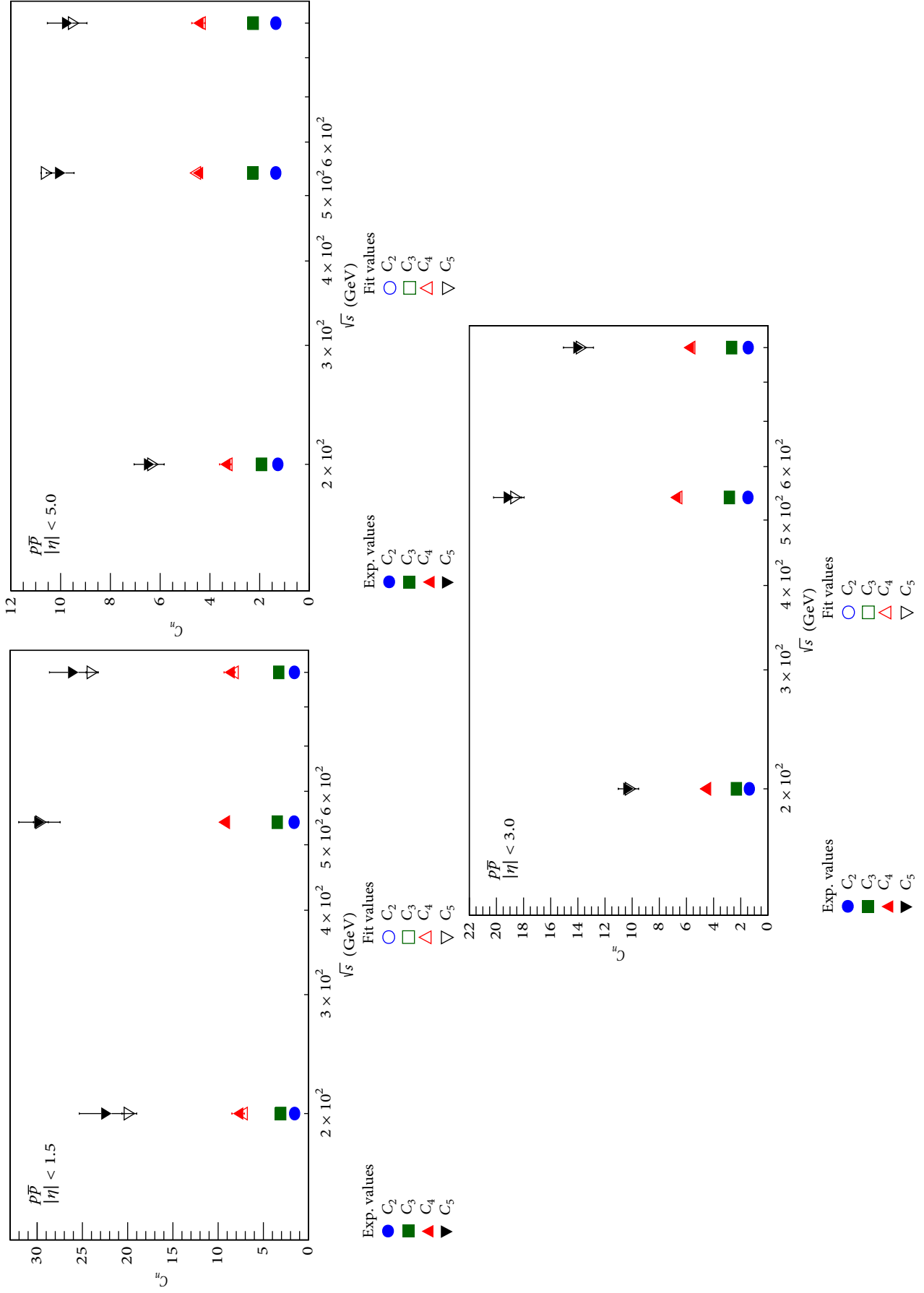


FIGURE 8. Normalized moments C_n of modified shifted Gompertz distribution for the $p\bar{p}$ collisions recorded by the UA5 experiment at different energies and in three pseudo-rapidity bins, $|\eta| < 1.5, 3.0$ and 5.0 .

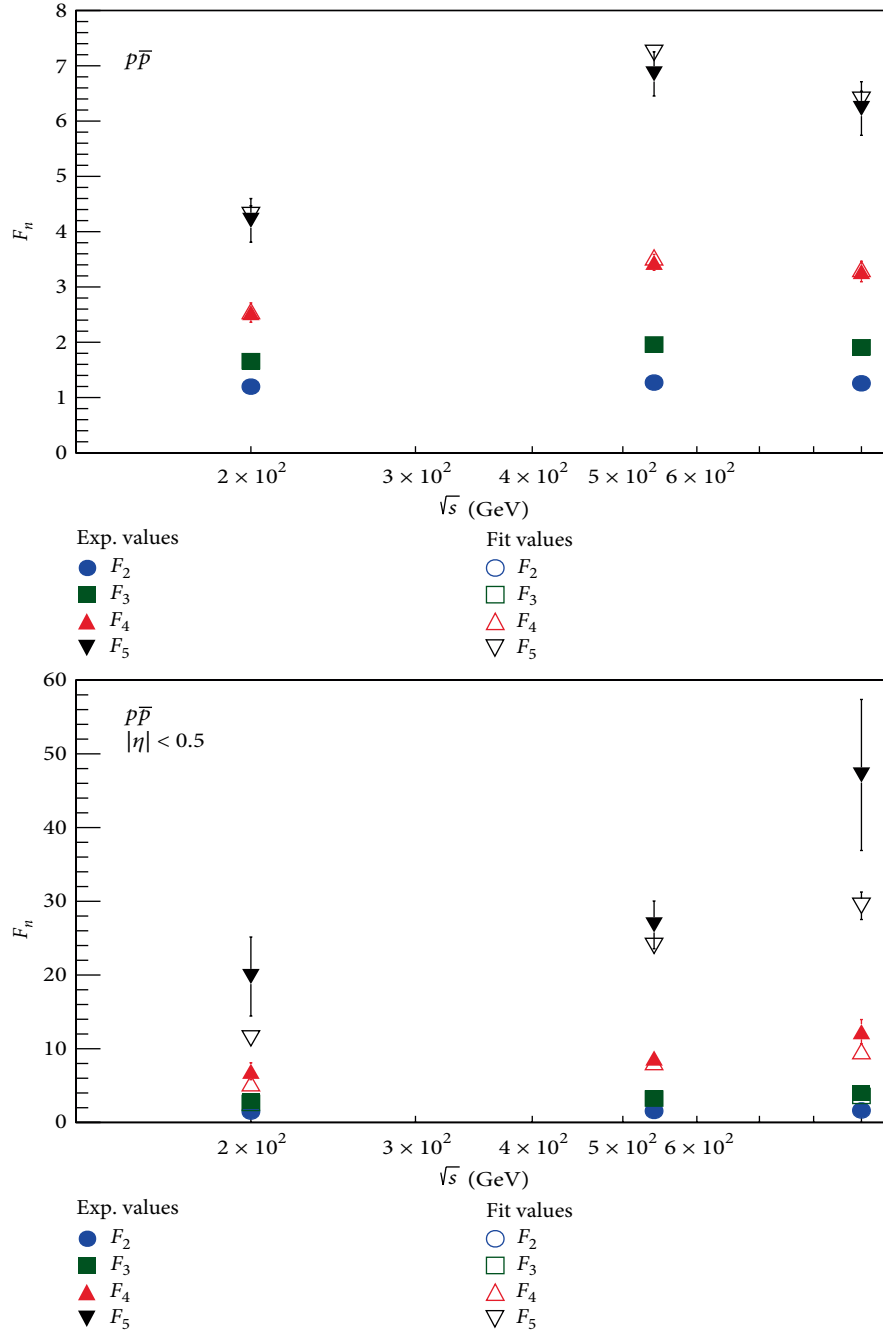


FIGURE 9: Normalized factorial moments F_n of modified shifted Gompertz distribution for the $p\bar{p}$ collisions recorded by the UA5 experiment at different energies, in full phase space and pseudo-rapidity bin, $|\eta| < 0.5$.

collisions and also from the fitted shifted Gompertz and modified shifted Gompertz distributions. The values of the moments are documented in Table 1.

Figures 7–10 show the normalized moments (C_q) and the normalized factorial moments (F_q) calculated from the data and from the modified shifted Gompertz distribution for the $p\bar{p}$ data at energies from 200, 540, and 900 GeV, in four rapidity bins and full phase space. The values of the moments are documented in Table 2.

Figures 11–14 show the normalized moments (C_q) and the normalized factorial moments (F_q) calculated from the data and from the modified shifted Gompertz distribution for the pp data at energies from 900, 2360, and 7000 GeV in five rapidity bins. The values of the moments are documented in Table 3.

It is observed that the moments decrease with the increase in rapidity window, at all energies. It is also interesting to compare the results from pp to $p\bar{p}$ collisions at the same c.m. energy. From the comparison at 900 GeV, we find that the moments have higher

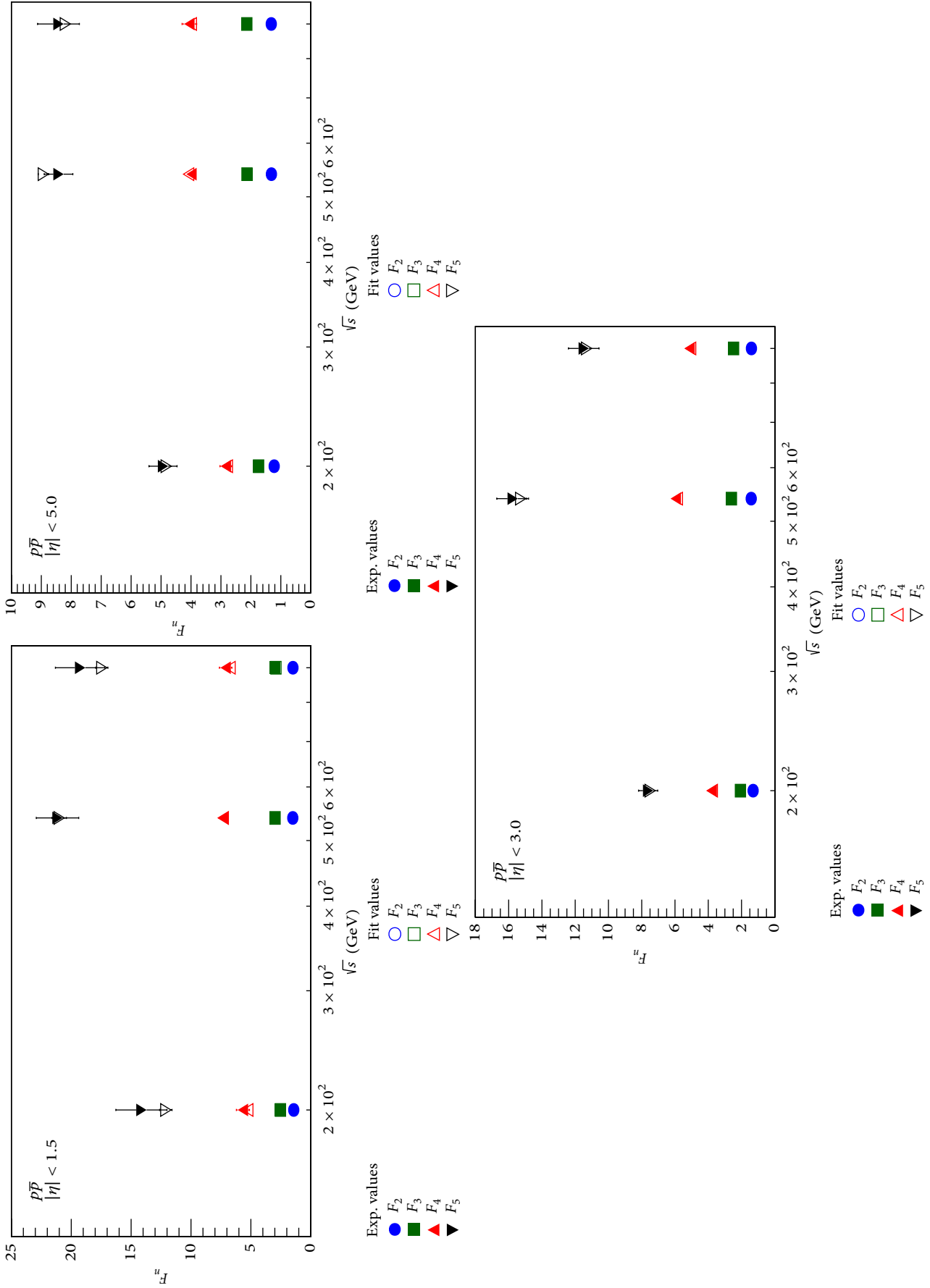


FIGURE 10: Normalized factorial moments F_n of modified shifted Gompertz distribution for the $p\bar{p}$ collisions recorded by the UA5 experiment at different energies and in three pseudo-rapidity bins, $|\eta| < 1.5, 3.0$ and 5.0 .

TABLE 2: Moments of experimental, shifted Gompertz and modified shifted Gompertz distributions for $p\bar{p}$ collisions.

Energy (GeV) ($p\bar{p}$)	$ \eta $	$\langle n \rangle$	Normalized moments (experiment)					Normalized factorial moments (experiment)				
			C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5		
200	0.5	2.501 ± 0.074	1.911 ± 0.053	4.895 ± 0.322	15.711 ± 1.876	60.718 ± 11.403	1.511 ± 0.046	2.923 ± 0.238	6.945 ± 1.147	19.796 ± 5.362		
	1.5	7.940 ± 0.206	1.556 ± 0.037	3.159 ± 0.167	7.792 ± 0.695	22.349 ± 3.004	1.430 ± 0.035	2.603 ± 0.144	5.664 ± 0.548	14.140 ± 2.136		
	3	15.540 ± 0.294	1.368 ± 0.024	2.329 ± 0.082	4.630 ± 0.249	10.275 ± 0.754	1.304 ± 0.024	2.073 ± 0.075	3.791 ± 0.209	7.615 ± 0.577		
	5	20.285 ± 0.479	1.278 ± 0.028	1.948 ± 0.087	3.379 ± 0.233	6.442 ± 0.605	1.229 ± 0.027	1.764 ± 0.080	2.836 ± 0.198	4.935 ± 0.470		
	Full	21.175 ± 0.500	1.244 ± 0.028	1.823 ± 0.082	3.023 ± 0.207	5.496 ± 0.510	1.197 ± 0.027	1.651 ± 0.075	2.536 ± 0.176	4.204 ± 0.395		
540	0.5	3.004 ± 0.047	1.915 ± 0.029	5.037 ± 0.175	16.684 ± 1.004	66.035 ± 6.085	1.582 ± 0.026	3.347 ± 0.134	8.737 ± 0.654	26.799 ± 3.240		
	1.5	9.471 ± 0.137	1.610 ± 0.022	3.480 ± 0.107	9.345 ± 0.482	29.750 ± 2.289	1.504 ± 0.021	2.992 ± 0.096	7.331 ± 0.403	21.149 ± 1.773		
	3	18.968 ± 0.252	1.474 ± 0.017	2.857 ± 0.073	6.815 ± 0.284	19.083 ± 1.127	1.422 ± 0.017	2.630 ± 0.068	5.955 ± 0.254	15.758 ± 0.956		
	5	26.330 ± 0.342	1.353 ± 0.016	2.271 ± 0.056	4.480 ± 0.176	10.016 ± 0.556	1.315 ± 0.015	2.120 ± 0.053	3.984 ± 0.159	8.425 ± 0.479		
	Full	28.375 ± 0.374	1.305 ± 0.016	2.079 ± 0.054	3.868 ± 0.159	8.128 ± 0.465	1.269 ± 0.016	1.943 ± 0.051	3.446 ± 0.143	6.853 ± 0.400		
900	0.5	3.614 ± 0.087	1.940 ± 0.046	5.410 ± 0.318	19.789 ± 2.173	89.311 ± 15.604	1.664 ± 0.042	3.952 ± 0.269	12.314 ± 1.648	47.134 ± 10.245		
	1.5	11.503 ± 0.247	1.582 ± 0.031	3.368 ± 0.144	8.736 ± 0.614	26.012 ± 2.634	1.496 ± 0.029	2.970 ± 0.131	7.107 ± 0.522	19.258 ± 2.066		
	3	21.925 ± 0.428	1.456 ± 0.027	2.695 ± 0.103	5.828 ± 0.341	13.956 ± 1.108	1.410 ± 0.026	2.500 ± 0.096	5.124 ± 0.303	11.487 ± 0.920		
	5	32.263 ± 0.642	1.354 ± 0.025	2.278 ± 0.088	4.464 ± 0.266	9.736 ± 0.797	1.323 ± 0.025	2.154 ± 0.083	4.054 ± 0.244	8.427 ± 0.698		
	Full	35.632 ± 0.686	1.284 ± 0.023	2.003 ± 0.073	3.606 ± 0.203	7.185 ± 0.554	1.256 ± 0.022	1.897 ± 0.069	3.279 ± 0.186	6.227 ± 0.485		
Energy (GeV) ($p\bar{p}$)	$ \eta $	$\langle n \rangle$	Normalized moments (shifted Gompertz)					Normalized factorial moments (shifted Gompertz)				
200	0.5	2.475 ± 0.029	1.912 ± 0.024	4.864 ± 0.122	15.015 ± 0.573	53.249 ± 2.762	1.508 ± 0.020	2.873 ± 0.078	6.261 ± 0.264	14.708 ± 0.873		
	1.5	7.917 ± 0.055	1.536 ± 0.011	3.018 ± 0.045	7.021 ± 0.158	18.455 ± 0.560	1.410 ± 0.011	2.468 ± 0.037	4.992 ± 0.115	11.123 ± 0.349		
	3	15.173 ± 0.061	1.381 ± 0.007	2.348 ± 0.022	4.626 ± 0.063	10.137 ± 0.181	1.315 ± 0.007	2.083 ± 0.020	3.762 ± 0.052	7.425 ± 0.134		
	5	20.152 ± 0.094	1.297 ± 0.006	2.014 ± 0.020	3.560 ± 0.053	6.915 ± 0.137	1.247 ± 0.006	1.826 ± 0.018	2.995 ± 0.045	5.314 ± 0.106		
	Full	21.064 ± 0.152	1.245 ± 0.010	1.827 ± 0.028	3.039 ± 0.070	5.560 ± 0.171	1.198 ± 0.009	1.654 ± 0.026	2.549 ± 0.059	4.254 ± 0.132		
540	0.5	2.976 ± 0.016	1.917 ± 0.012	5.082 ± 0.064	17.035 ± 0.324	67.968 ± 1.738	1.581 ± 0.010	3.376 ± 0.045	8.940 ± 0.182	27.480 ± 0.756		
	1.5	9.428 ± 0.044	1.599 ± 0.008	3.421 ± 0.035	9.095 ± 0.143	28.618 ± 0.611	1.493 ± 0.008	2.935 ± 0.031	7.109 ± 0.115	20.226 ± 0.444		
	3	18.774 ± 0.076	1.448 ± 0.006	2.705 ± 0.023	6.155 ± 0.081	16.359 ± 0.293	1.394 ± 0.006	2.479 ± 0.021	5.335 ± 0.071	13.338 ± 0.243		
	5	26.154 ± 0.093	1.345 ± 0.005	2.258 ± 0.017	4.523 ± 0.053	10.440 ± 0.166	1.307 ± 0.005	2.107 ± 0.016	4.026 ± 0.048	8.822 ± 0.141		
	Full	28.112 ± 0.084	1.307 ± 0.004	2.099 ± 0.014	3.982 ± 0.040	8.625 ± 0.115	1.271 ± 0.004	1.962 ± 0.013	3.551 ± 0.036	7.299 ± 0.099		
900	0.5	3.555 ± 0.028	1.894 ± 0.016	4.905 ± 0.087	15.756 ± 0.424	59.066 ± 2.159	1.613 ± 0.015	3.465 ± 0.065	8.991 ± 0.259	26.371 ± 1.043		
	1.5	11.352 ± 0.054	1.557 ± 0.008	3.187 ± 0.032	7.850 ± 0.121	22.057 ± 0.460	1.469 ± 0.007	2.791 ± 0.029	6.294 ± 0.099	15.956 ± 0.342		
	3	21.835 ± 0.075	1.441 ± 0.005	2.604 ± 0.019	5.486 ± 0.060	12.867 ± 0.189	1.395 ± 0.005	2.410 ± 0.018	4.803 ± 0.053	10.539 ± 0.156		
	5	32.073 ± 0.101	1.324 ± 0.004	2.134 ± 0.014	3.980 ± 0.040	8.275 ± 0.111	1.293 ± 0.004	2.013 ± 0.013	3.595 ± 0.036	7.105 ± 0.096		
	Full	35.567 ± 0.157	1.268 ± 0.006	1.935 ± 0.018	3.406 ± 0.048	6.680 ± 0.125	1.240 ± 0.006	1.830 ± 0.017	3.091 ± 0.043	5.774 ± 0.109		

TABLE 2: Continued.

Energy (GeV) ($\overline{p\overline{p}}$)	$ \eta $	$\langle n \rangle$	Normalized moments (modified shifted Gompertz)					Normalized factorial moments (modified shifted Gompertz)				
			C_2	C_3	C_4	C_5		F_2	F_3	F_4	F_5	
200	0.5	2.477 ± 0.041	1.881 ± 0.033	4.604 ± 0.166	13.493 ± 0.742	45.166 ± 3.376	1.478 ± 0.028	2.651 ± 0.104	5.320 ± 0.326	11.404 ± 0.993		
	1.5	7.895 ± 0.071	1.547 ± 0.016	3.085 ± 0.063	7.340 ± 0.226	19.818 ± 0.819	1.420 ± 0.015	2.529 ± 0.053	5.257 ± 0.167	12.101 ± 0.520		
	3	15.537 ± 0.107	1.367 ± 0.010	2.321 ± 0.034	4.593 ± 0.101	10.130 ± 0.300	1.302 ± 0.009	2.065 ± 0.030	3.758 ± 0.084	7.493 ± 0.226		
	5	20.484 ± 0.134	1.270 ± 0.009	1.923 ± 0.026	3.317 ± 0.068	6.293 ± 0.172	1.221 ± 0.008	1.742 ± 0.024	2.786 ± 0.057	4.827 ± 0.134		
	Full	21.124 ± 0.185	1.245 ± 0.012	1.830 ± 0.036	3.054 ± 0.089	5.610 ± 0.216	1.197 ± 0.012	1.657 ± 0.033	2.565 ± 0.075	4.301 ± 0.167		
540	0.5	2.981 ± 0.022	1.901 ± 0.016	4.929 ± 0.085	15.986 ± 0.419	61.449 ± 2.190	1.566 ± 0.014	3.241 ± 0.059	8.192 ± 0.234	23.951 ± 0.945		
	1.5	9.463 ± 0.055	1.606 ± 0.011	3.468 ± 0.047	9.313 ± 0.192	29.587 ± 0.828	1.501 ± 0.010	2.981 ± 0.041	7.305 ± 0.154	21.009 ± 0.607		
	3	18.863 ± 0.092	1.470 ± 0.008	2.836 ± 0.031	6.714 ± 0.112	18.569 ± 0.421	1.417 ± 0.007	2.608 ± 0.028	5.857 ± 0.099	15.278 ± 0.353		
	5	26.179 ± 0.113	1.352 ± 0.006	2.282 ± 0.022	4.582 ± 0.067	10.568 ± 0.207	1.314 ± 0.006	2.130 ± 0.021	4.080 ± 0.060	8.931 ± 0.177		
	Full	28.144 ± 0.100	1.306 ± 0.005	2.096 ± 0.017	3.964 ± 0.048	8.553 ± 0.138	1.271 ± 0.005	1.959 ± 0.016	3.535 ± 0.043	7.234 ± 0.118		
900	0.5	3.578 ± 0.038	1.907 ± 0.022	5.041 ± 0.120	16.619 ± 0.631	63.995 ± 3.464	1.627 ± 0.019	3.599 ± 0.093	9.671 ± 0.412	29.394 ± 1.868		
	1.5	11.428 ± 0.071	1.575 ± 0.010	3.303 ± 0.044	8.345 ± 0.172	23.917 ± 0.675	1.487 ± 0.010	2.905 ± 0.039	6.740 ± 0.142	17.448 ± 0.511		
	3	22.026 ± 0.098	1.449 ± 0.007	2.672 ± 0.026	5.755 ± 0.084	13.741 ± 0.266	1.404 ± 0.007	2.479 ± 0.024	5.060 ± 0.074	11.315 ± 0.221		
	5	32.341 ± 0.129	1.350 ± 0.006	2.259 ± 0.019	4.393 ± 0.056	9.482 ± 0.163	1.319 ± 0.006	2.136 ± 0.018	3.988 ± 0.052	8.197 ± 0.142		
	Full	35.734 ± 0.199	1.285 ± 0.008	2.015 ± 0.024	3.653 ± 0.066	7.354 ± 0.177	1.257 ± 0.008	1.908 ± 0.023	3.326 ± 0.060	6.386 ± 0.155		

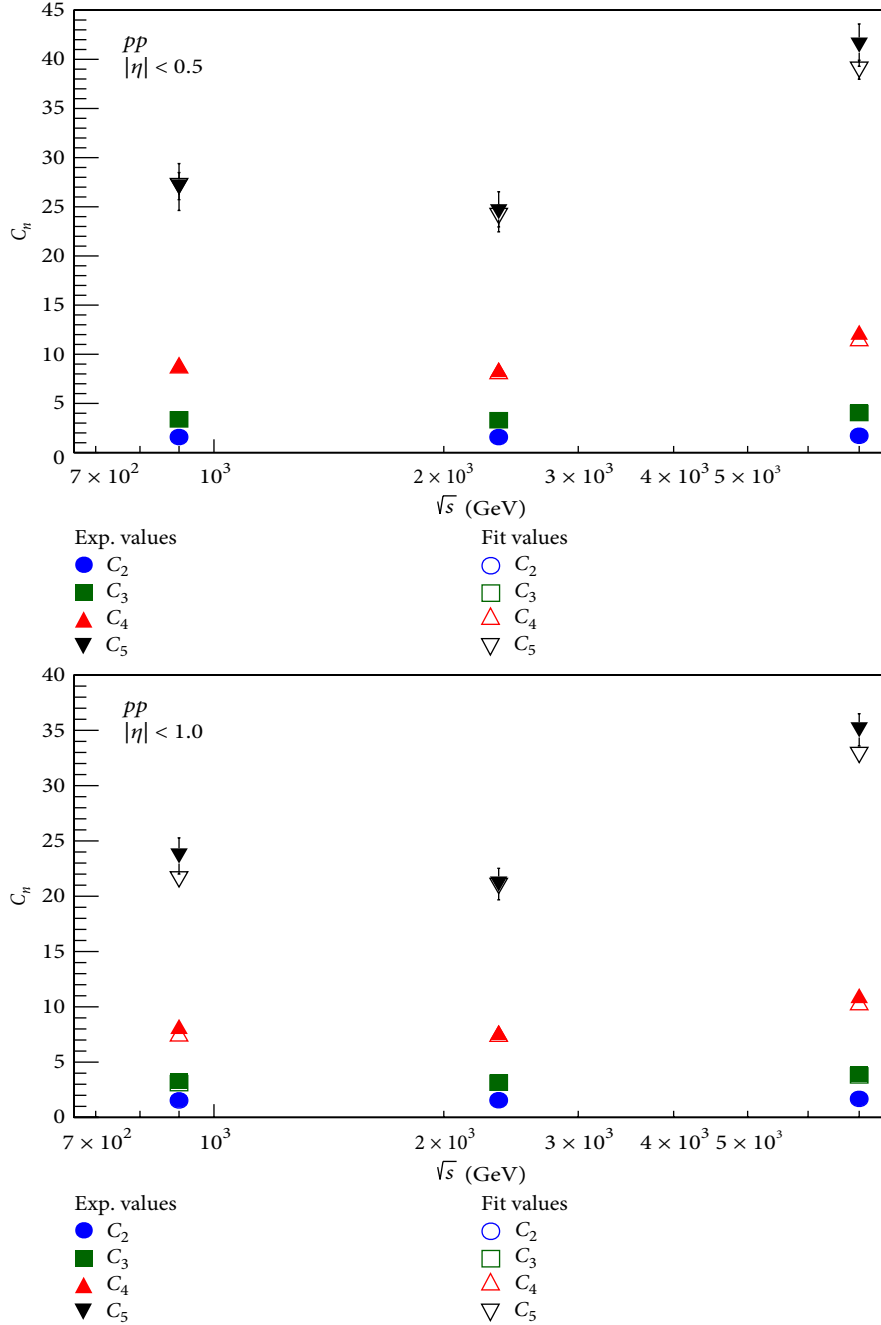


FIGURE 11: Normalized moments C_n of modified shifted Gompertz distribution for the pp collisions recorded by the CMS experiment at different energies and in two pseudo-rapidity bins, $|\eta| < 0.5$ and 1.0 .

values in case of $p\bar{p}$ than pp . For example, we include Figure 15 to show the dependence for $|\eta| < 0.5$. However values of multiplicity for pp collisions at 900 GeV in full phase space are not available. In addition, for pp and for $p\bar{p}$ the rapidity intervals are also different. So a direct comparison is not feasible.

Figures 5–14, described above, are shown only for the modified shifted Gompertz distributions. To avoid repetition and cluttering of figures, the figures for shifted Gompertz distributions are only included for e^+e^- collisions and are not included for pp and $p\bar{p}$ data. However the values of the moments are given in Tables 1–3.

The predictions for normalized moments and normalized factorial moments are also made for e^+e^- collisions at 500 GeV at a future collider. Using the shifted Gompertz distribution, the prediction for probability distribution is made, as shown in Figure 16. Using this predicted distribution, 1σ confidence interval band, moments have been calculated, as given in Table 4.

It is observed that in case of $p\bar{p}$ and pp interactions, C_2 , C_3 and F_2 , F_3 remain roughly constant with energy while higher moments C_4 , C_5 , F_4 , F_5 show an increase with increasing energy. The increase becomes more evident for larger rapidity windows. This leads to the depiction of violation of KNO scaling at high

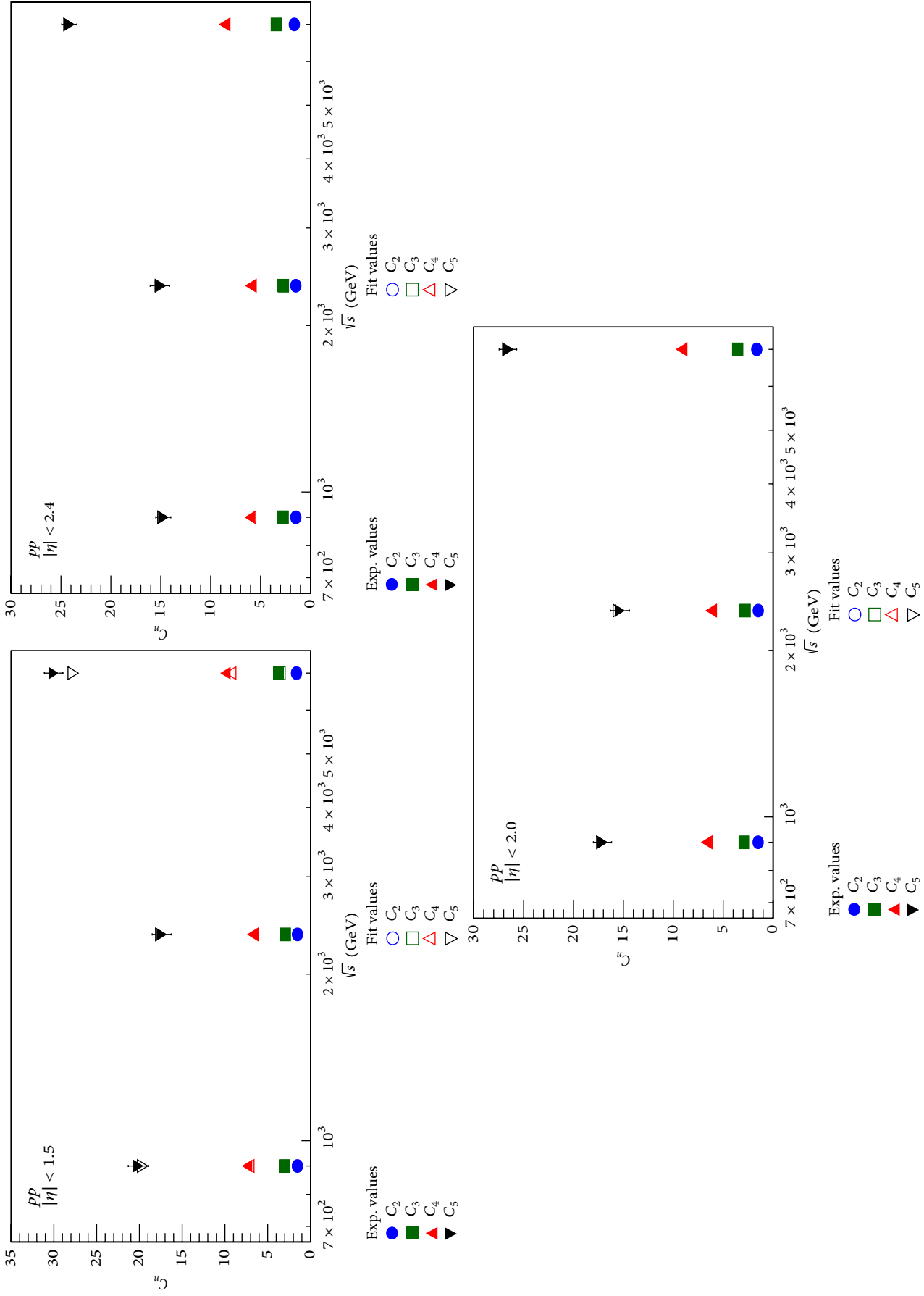


FIGURE 12: Normalized moments C_n of modified shifted Gompertz distribution for the pp collisions recorded by the CMS experiment at different energies and in three pseudo-rapidity bins, $|\eta| < 1.5$, 2.0 and 2.4 .

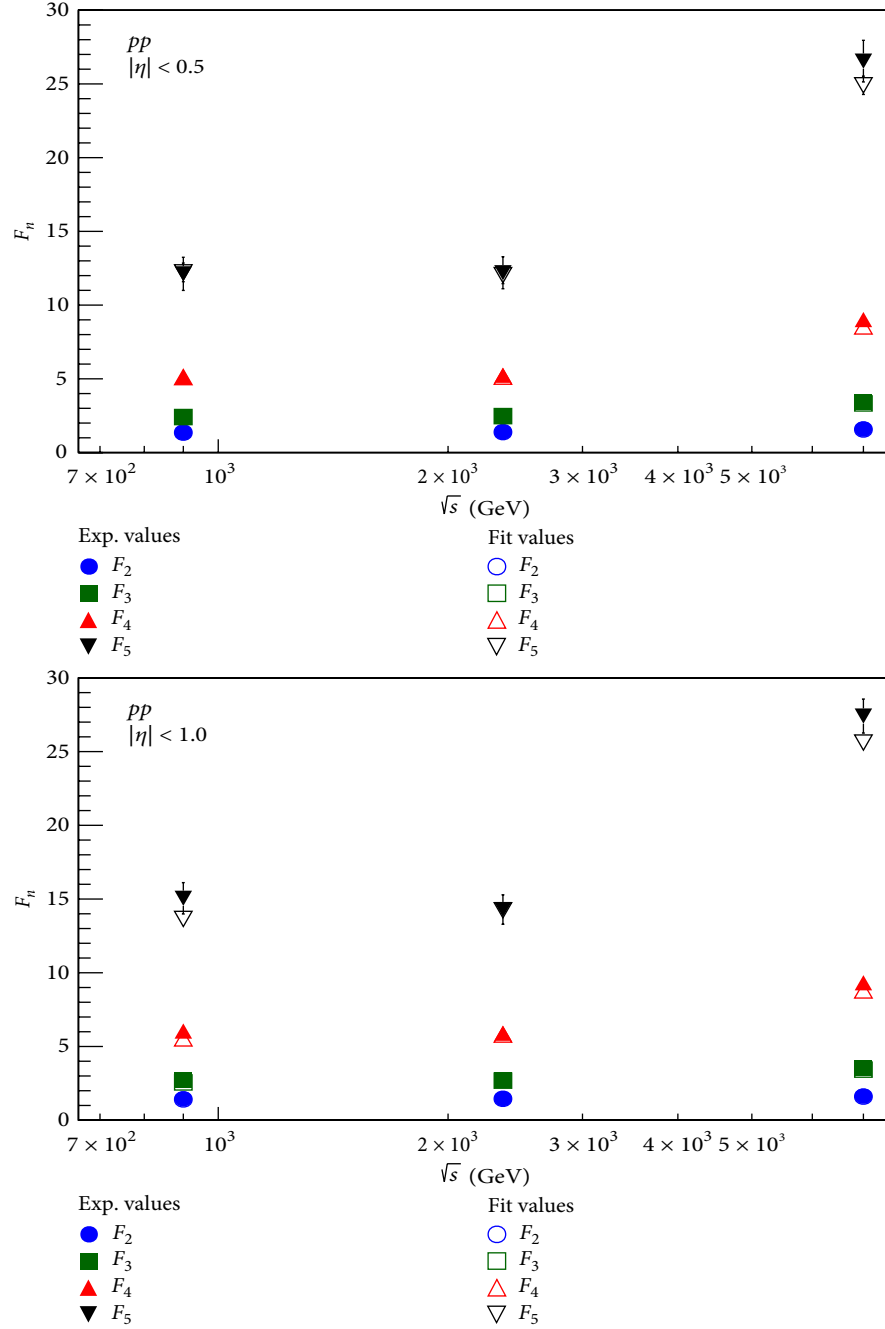


FIGURE 13: Normalized factorial moments F_n of modified shifted Gompertz distribution for the pp collisions recorded by the CMS experiment at different energies and in two pseudo-rapidity bins, $|\eta| < 0.5$ and 1.0 .

energies. Same conclusions have been reported in Ref. [21] for pp collisions at the LHC. However for the e^+e^- collisions, the moments are roughly independent of energy. This is expected as the collision energy is low, nearly at the onset of energy range, which marks the start of KNO scaling violation.

4. Uncertainties on Moments

Given a distribution $P(n)$ which is normalized to unity with an uncertainty ϵ_n , and assuming that the errors on the

individual bins are uncorrelated, the moment errors can be calculated using the method described in [22], using the partial derivatives:

$$\frac{\partial C_q}{\partial P_n} = \frac{n^q \langle n \rangle - \langle n^q \rangle q n}{\langle n \rangle^{q+1}}. \quad (13)$$

$$\frac{\partial F_q}{\partial P_n} = \frac{n(n-1)\dots(n-q+1)\langle n \rangle - \langle n(n-1)\dots(n-q+1) \rangle q n}{\langle n \rangle^{q+1}}. \quad (14)$$

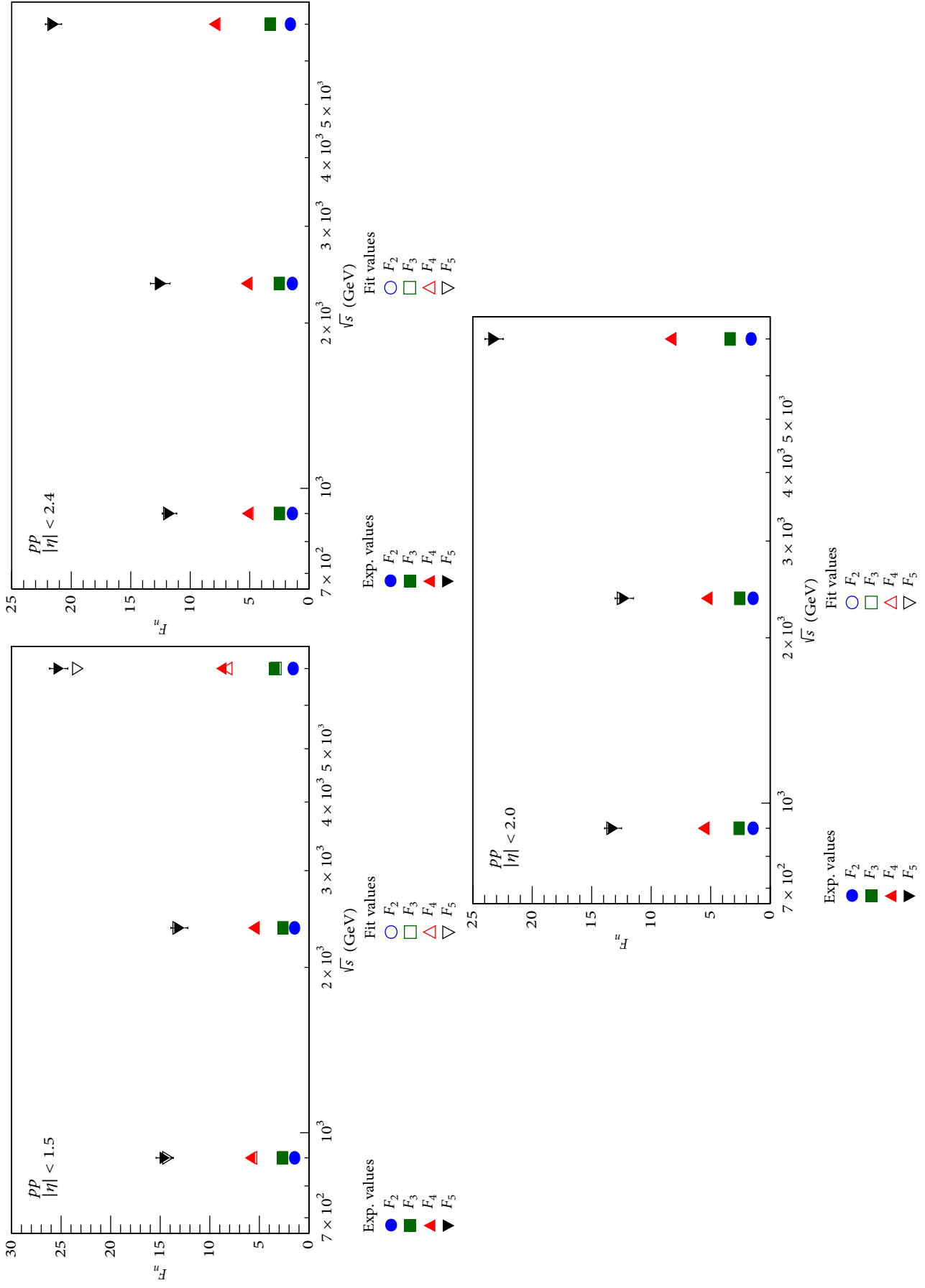


FIGURE 14: Normalized factorial moments F_n of modified shifted Gompertz distribution for the pp collisions recorded by the CMS experiment at different energies and in three pseudo-rapidity bins, $|\eta| < 1.5$, 2.0 and 2.4.

TABLE 3: Moments of experimental, shifted Gompertz and modified shifted Gompertz distributions for pp collisions.

Energy (GeV) (pp)	$ \eta $	$\langle n \rangle$	Normalized moments (experiment)					Normalized factorial moments (experiment)				
			C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5		
900	0.5	4.426±0.084	1.582±0.036	3.386±0.152	8.893±0.592	27.013±2.387	1.356±0.032	2.416±0.113	5.122±0.356	12.119±1.125		
	1.0	8.059±0.121	1.570±0.029	3.272±0.117	8.229±0.432	23.642±1.636	1.446±0.027	2.719±0.099	6.047±0.324	15.050±1.064		
	1.5	11.744±0.151	1.538±0.024	3.092±0.094	7.413±0.331	20.116±1.188	1.453±0.023	2.714±0.083	5.952±0.269	14.542±0.874		
	2.0	15.507±0.183	1.504±0.022	2.912±0.080	6.659±0.270	17.095±0.914	1.440±0.021	2.630±0.073	5.599±0.229	13.205±0.714		
	2.4	18.432±0.211	1.474±0.020	2.759±0.073	6.042±0.235	14.748±0.755	1.420±0.020	2.525±0.067	5.190±0.203	11.742±0.607		
2360	0.5	5.289±0.095	1.585±0.034	3.334±0.138	8.443±0.521	24.490±2.041	1.396±0.031	2.507±0.107	5.243±0.337	12.194±1.086		
	1.0	9.722±0.144	1.565±0.027	3.185±0.106	7.689±0.383	21.099±1.426	1.463±0.026	2.723±0.092	5.900±0.300	14.287±0.998		
	1.5	13.968±0.196	1.523±0.025	2.956±0.093	6.738±0.316	17.392±1.114	1.451±0.024	2.640±0.084	5.552±0.264	13.072±0.858		
	2.0	18.323±0.251	1.490±0.023	2.806±0.085	6.177±0.282	15.347±0.955	1.435±0.023	2.568±0.079	5.306±0.245	12.256±0.776		
	2.4	21.696±0.298	1.471±0.023	2.736±0.084	6.008±0.280	15.092±0.973	1.425±0.022	2.537±0.078	5.285±0.249	12.518±0.823		
7000	0.5	6.924±0.077	1.732±0.026	4.159±0.114	12.245±0.486	41.444±2.156	1.588±0.024	3.450±0.097	9.020±0.366	26.546±1.413		
	1.0	13.122±0.118	1.705±0.021	3.945±0.088	11.020±0.353	35.044±1.461	1.629±0.020	3.567±0.081	9.323±0.301	27.410±1.154		
	1.5	19.421±0.154	1.669±0.018	3.725±0.073	9.946±0.279	30.027±1.094	1.617±0.018	3.472±0.069	8.843±0.249	25.240±0.926		
	2.0	25.870±0.188	1.640±0.016	3.562±0.064	9.187±0.234	26.573±0.879	1.601±0.016	3.375±0.061	8.388±0.215	23.203±0.771		
	2.4	30.790±0.213	1.616±0.015	3.433±0.059	8.613±0.209	24.169±0.760	1.584±0.015	3.277±0.056	7.963±0.194	21.495±0.678		
900	0.5	4.433±0.033	1.571±0.017	3.348±0.065	8.804±0.245	26.927±0.973	1.345±0.015	2.387±0.048	5.083±0.146	12.190±0.453		
	1.0	8.099±0.038	1.522±0.010	3.056±0.038	7.474±0.132	21.215±0.482	1.399±0.010	2.522±0.032	5.455±0.097	13.480±0.310		
	1.5	11.768±0.038	1.521±0.007	3.019±0.025	7.213±0.086	19.783±0.307	1.437±0.007	2.645±0.022	5.791±0.070	14.371±0.226		
	2.0	15.536±0.040	1.499±0.005	2.887±0.019	6.604±0.062	17.122±0.209	1.435±0.005	2.606±0.017	5.556±0.052	13.271±0.163		
	2.4	18.445±0.043	1.470±0.004	2.737±0.016	5.996±0.049	14.790±0.159	1.416±0.004	2.504±0.014	5.152±0.043	11.810±0.128		
2360	0.5	5.275±0.035	1.580±0.015	3.318±0.056	8.439±0.206	24.846±0.798	1.390±0.013	2.491±0.043	5.249±0.132	12.514±0.422		
	1.0	9.608±0.040	1.570±0.009	3.206±0.034	7.845±0.119	22.148±0.447	1.466±0.008	2.738±0.029	6.023±0.093	15.112±0.314		
	1.5	13.762±0.045	1.536±0.007	3.013±0.024	7.000±0.082	18.718±0.297	1.464±0.006	2.688±0.022	5.774±0.069	14.160±0.231		
	2.0	17.994±0.051	1.512±0.005	2.884±0.020	6.472±0.065	16.550±0.224	1.456±0.005	2.639±0.018	5.560±0.056	13.253±0.183		
	2.4	21.374±0.055	1.486±0.005	2.795±0.017	6.276±0.058	16.423±0.207	1.439±0.005	2.591±0.016	5.527±0.052	13.694±0.176		
7000	0.5	7.178±0.028	1.652±0.010	3.749±0.040	10.568±0.158	34.861±0.668	1.513±0.009	3.098±0.034	7.771±0.118	22.472±0.435		
	1.0	13.744±0.032	1.615±0.006	3.488±0.022	9.202±0.082	28.179±0.321	1.542±0.006	3.146±0.020	7.771±0.070	22.099±0.253		
	1.5	20.066±0.034	1.609±0.004	3.426±0.016	8.811±0.056	26.028±0.213	1.560±0.004	3.191±0.015	7.829±0.050	21.925±0.180		
	2.0	25.909±0.034	1.642±0.003	3.559±0.012	9.216±0.045	27.029±0.169	1.604±0.003	3.371±0.012	8.419±0.041	23.653±0.148		
	2.4	30.658±0.034	1.628±0.003	3.471±0.010	8.794±0.037	25.165±0.136	1.596±0.003	3.313±0.010	8.134±0.034	22.423±0.122		

TABLE 3: Continued.

Energy (GeV) (pp)	$ \eta $	$\langle n \rangle$	Normalized moments (modified shifted Gompertz)					Normalized factorial moments (modified shifted Gompertz)				
			C_2	C_3	C_4	C_5		F_2	F_3	F_4	F_5	
900	0.5	4.423 ± 0.046	1.579 ± 0.024	3.376 ± 0.094	8.878 ± 0.352	27.104 ± 1.379	1.353 ± 0.022	2.407 ± 0.069	5.117 ± 0.207	12.219 ± 0.634		
	1.0	8.097 ± 0.048	1.534 ± 0.014	3.101 ± 0.049	7.601 ± 0.171	21.522 ± 0.618	1.411 ± 0.013	2.563 ± 0.042	5.549 ± 0.126	13.651 ± 0.395		
	1.5	11.784 ± 0.051	1.521 ± 0.010	3.020 ± 0.035	7.199 ± 0.117	19.628 ± 0.410	1.436 ± 0.009	2.647 ± 0.031	5.778 ± 0.095	14.234 ± 0.300		
	2.0	15.510 ± 0.054	1.497 ± 0.008	2.891 ± 0.027	6.624 ± 0.087	17.169 ± 0.290	1.433 ± 0.007	2.609 ± 0.024	5.573 ± 0.074	13.299 ± 0.226		
	2.4	18.433 ± 0.059	1.465 ± 0.007	2.732 ± 0.023	5.995 ± 0.072	14.792 ± 0.228	1.411 ± 0.007	2.500 ± 0.021	5.152 ± 0.062	11.809 ± 0.183		
2360	0.5	5.309 ± 0.050	1.575 ± 0.021	3.290 ± 0.080	8.292 ± 0.292	24.062 ± 1.111	1.386 ± 0.019	2.471 ± 0.062	5.148 ± 0.186	12.032 ± 0.579		
	1.0	9.746 ± 0.052	1.554 ± 0.012	3.141 ± 0.044	7.570 ± 0.151	20.928 ± 0.546	1.452 ± 0.011	2.684 ± 0.038	5.810 ± 0.117	14.237 ± 0.380		
	1.5	13.978 ± 0.062	1.516 ± 0.009	2.937 ± 0.033	6.711 ± 0.108	17.510 ± 0.374	1.445 ± 0.009	2.622 ± 0.030	5.533 ± 0.090	13.208 ± 0.288		
	2.0	18.284 ± 0.069	1.486 ± 0.008	2.797 ± 0.027	6.183 ± 0.085	15.515 ± 0.283	1.431 ± 0.007	2.559 ± 0.025	5.313 ± 0.074	12.414 ± 0.230		
	2.4	21.760 ± 0.079	1.463 ± 0.007	2.710 ± 0.024	5.949 ± 0.078	15.039 ± 0.263	1.417 ± 0.007	2.513 ± 0.022	5.235 ± 0.069	12.498 ± 0.221		
7000	0.5	6.979 ± 0.035	1.707 ± 0.013	4.024 ± 0.056	11.645 ± 0.226	38.944 ± 0.966	1.564 ± 0.013	3.331 ± 0.047	8.554 ± 0.168	24.908 ± 0.623		
	1.0	13.220 ± 0.041	1.677 ± 0.008	3.800 ± 0.033	10.424 ± 0.125	32.771 ± 0.502	1.601 ± 0.008	3.431 ± 0.030	8.802 ± 0.106	25.612 ± 0.394		
	1.5	19.665 ± 0.047	1.634 ± 0.006	3.561 ± 0.024	9.316 ± 0.086	27.750 ± 0.324	1.583 ± 0.006	3.317 ± 0.022	8.275 ± 0.076	23.324 ± 0.273		
	2.0	25.817 ± 0.048	1.636 ± 0.005	3.547 ± 0.018	9.152 ± 0.065	26.588 ± 0.240	1.597 ± 0.005	3.360 ± 0.017	8.354 ± 0.059	23.225 ± 0.210		
	2.4	30.731 ± 0.050	1.612 ± 0.004	3.417 ± 0.015	8.582 ± 0.054	24.209 ± 0.193	1.579 ± 0.004	3.262 ± 0.015	7.934 ± 0.050	21.540 ± 0.172		

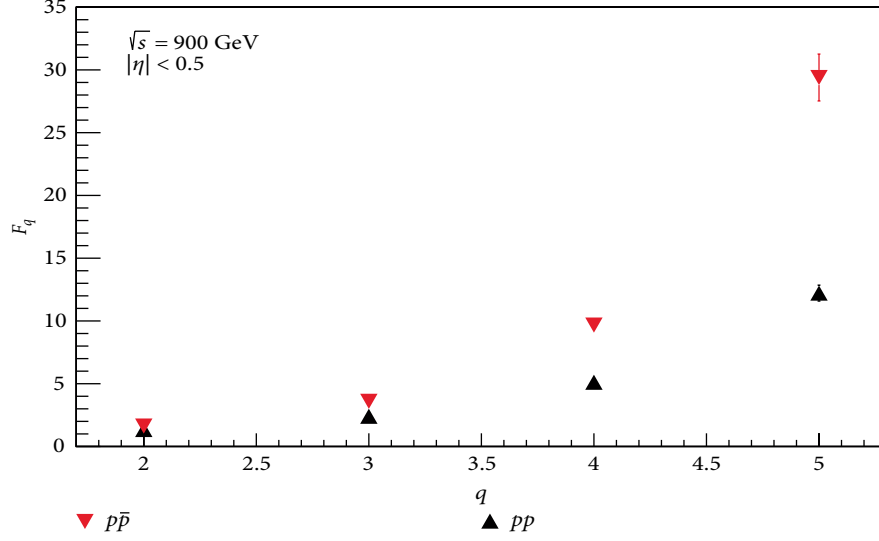


FIGURE 15: Comparison of normalised factorial moments for pp and $p\bar{p}$ interactions at the same c.m. energy in the same rapidity window.

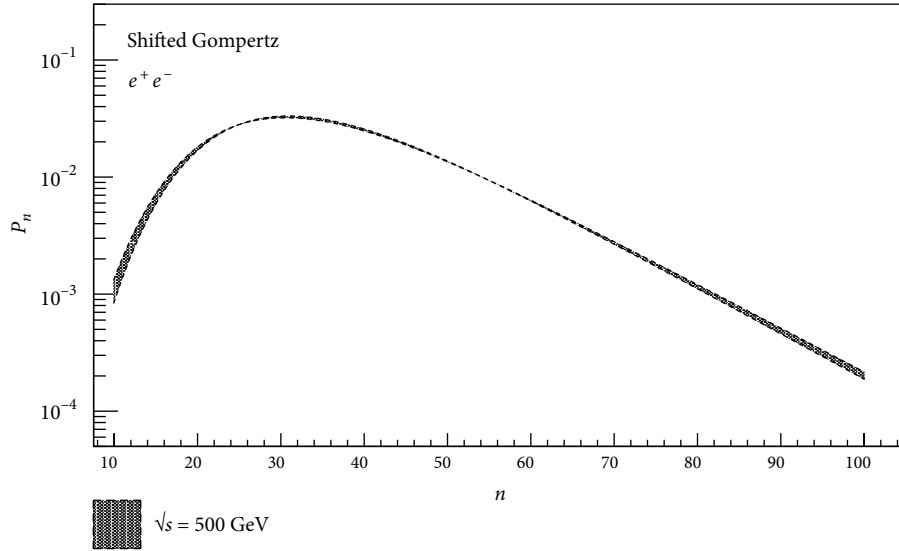


FIGURE 16: Probability distribution predicted from shifted Gompertz distribution, in 1σ confidence interval band, for the e^+e^- collisions at 500 GeV.

The total error is then

$$E_q^2 = \sum_n \left(\frac{\partial X_q}{\partial P_n} \epsilon_n \right)^2, \quad (15)$$

where X_q is C_q or F_q .

In the published data, multiplicities are given either as (value + statistical error + systematic error) or as (value + error). The errors on the multiplicities have been taken as the total error, by adding the statistical and systematic errors in quadrature.

5. Conclusion

An analysis of moments of multiplicity distributions described within a newly proposed statistical distribution, the shifted

Gompertz distribution and its modified form has been done. We had proposed and shown that the use of this statistical distribution for studying the multiplicity distributions in high energy collisions reproduces the results in e^+e^- , $p\bar{p}$, and pp collisions very well.

A good agreement between the normalized moments as well as normalized factorial moments obtained from the shifted Gompertz distribution and its modified form with the experimental values, serves as a good test of the validity of the proposed distribution. The results have reproduced the violation of KNO scaling as observed for higher moments in the measured data. At higher energies, particularly at LHC energies, the moments strongly are dependent on the energy. The information dissemination from such an analysis is often used to study the patterns and correlations

TABLE 4: Predicted moments at $\sqrt{s} = 500$ GeV from shifted Gompertz distribution for e^+e^- collisions.

Energy (GeV) (e^+e^-)	$\langle n \rangle$	Normalized moments (predicted)					Normalized factorial moments (predicted)				
		C_2	C_3	C_4	C_5		F_2	F_3	F_4	F_5	
500	37.081 ± 0.087	1.145 ± 0.003	1.487 ± 0.008	2.167 ± 0.018	3.496 ± 0.038		1.118 ± 0.003	1.396 ± 0.008	1.936 ± 0.016	2.948 ± 0.032	

in the multiparticle final state of high-energy collisions in the presence of statistical fluctuations. In this connection, we find that the factorial moments are large indicating correlations amongst the produced particles. The predictions for normalized moments and normalized factorial moments are also made for e^+e^- collisions at 500 GeV at a future collider.

Data Availability

All the data used in the paper can be obtained from the references quoted or from the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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