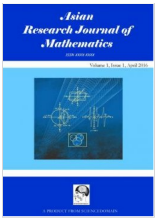

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Dissipation of Solutions for Linear Plate Equations

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Authors' contributions

This work was carried out in collaboration between both authors. Author XC designed the study, wrote the protocol, and wrote the first draft of the manuscript. Author YL managed the analyses of the study and revised the manuscript. Both authors read and approved the final manuscript.

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Short Research Article

Abstract

Aims/ Objectives: To study the dissipation of solutions for a plate equation
Methodology: energy method, Fourier analysis.
Results: The energy estimate and decay estimates of solutions are obtained.
Conclusion: The dissipative term has effect on the decay of solutions and the dissipative structure is not of regularity-loss type.

Keywords: linear plate equation; point-wise estimate in the Fourier space; decay estimates; energy estimates.

2010 Mathematics Subject Classification: 35B40; 35L30

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1 Introduction

In this paper we consider the initial-value problem of the following linear plate equation in multi-dimensional space $\mathbb{R}^n$ with $n \geq 1$:

$$u_{tt} + (1 + \triangle^2)u - \triangle u_t = 0 \tag{1.1}$$

with the initial data

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x). \tag{1.2}$$

Here $u = u(x, t)$ is the unknown function of $x = (x_1, \cdots, x_n) \in \mathbb{R}^n$ and $t > 0$, and represents the transversal displacement of the plate at the point x and t. The decay structure is characterized by the property

$$\rho(\xi) = \frac{|\xi|^2}{1 + |\xi|^2},$$

where $\rho(\xi)$ is introduced in the point-wise estimate in the Fourier space of solutions to the linear problem. This $\rho(\xi)$ determines that the energy restricted in the lower-frequency domain decays polynomially and exponentially in the higher-frequency domain and the decay property is of standard type instead of regularity-loss type.

Equations of the fourth-order appear in problems of solid mechanics and in the theory of thin plates and beams, and elliptic equations of the fourth-order appear in some formulations of problems related to the Navier-Stokes equations as discussed in [1]. Da-Luz and Charão studied a semi-linear dissipative plate equation whose linear part is given by:

$$u_{tt} - \Delta u_{tt} + \Delta^2 u + u_t = 0. \tag{1.3}$$

They proved the global existence of solutions and a polynomial decay of the energy by exploiting an energy method in [2]. However their result was restricted to the lower dimensional case $1 \leq n \leq 5$. This restriction on the space dimension was removed by Sugitani and Kawashima by making use of the sharp decay estimates for the equation (1.3) in [3]. Liu and Wang studied the point-wise estimate of solutions to a dissipative wave equation

$$u_{tt} - \Delta u + u_t = 0, \tag{1.4}$$

and they showed that the decay structure in (1.4) is characterized by the function $\rho(\xi)$ in [4]. Although the two equations (1.1) and (1.4) have different orders, they have the similar decay structure, which is a point worthy to be mentioned. For more studies of such decay structure, we refer to [5, 6, 7, 8, 9]. For more studies on aspects of dissipation of plate equations, we refer to [10, 11, 12, 13]. Also, as for the study of decay properties for hyperbolic systems of memory-type dissipation, we refer to [14, 15, 16, 17, 18].

The main purpose of this paper is to study decay estimates of the initial-value problem (1.1) and (1.2). By using Fourier transform, We obtain the solution u to the linear problem (1.1) and (1.2) given by (2.3) and the solution operators $G(t)*$ and $H(t)*$. Moreover by employing the energy method in the Fourier space, we obtain the point-wise estimate in the Fourier space of solutions to the linear problem (1.1) and (1.2).

The contents of the paper are as follows. In section 2, Solution formula are obtained. Also, we obtain the estimates and properties of solutions operators, which is based on the point-wise estimate in the Fourier space of solutions to the linear problem. In section 3, we prove the decay estimates of solutions to the linear problem by virtue of the properties of solution operator. In section 4, we give the conclusion.

Before closing this section, we give some notations to be used below. Let $\mathcal{F}[f]$ denote the Fourier transform of f defined by

$$\mathcal{F}[f] = \hat{f}(\xi) := \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n} e^{-ix\cdot\xi} f(x)dx.$$

$L^p = L^p(\mathbb{R}^n)(1 \leq p \leq \infty)$ is the usual Lebesgue space with the norm $\|\cdot\|_{L^p}$. $W^{m,p}(\mathbb{R}^n)$, $m \in Z_+$,$p \in [1, \infty)$ denote the usual Sobolev space with its norm

$$\|f\|_{W^{m,p}} := (\sum_{k=0}^{m} \|\partial_x^k f\|_{L^p}^p)^{\frac{1}{p}}.$$

In particular, we use $W^{m,2} = H^m$. Here, for a nonnegative interger k, ∂_x^k denotes the totality or each of all the k-th order derivatives with respect to $x \in \mathbb{R}^n$. Also, $C^k(I; H^m(\mathbb{R}^n))$ denotes the space of k-times continuously differentiable functions on the interval I with values in the Sobolev space $H^m = H^m(\mathbb{R}^n)$. Finally, in this paper, we denote every positive constant by the same symbol C or c without confusion.

2 Energy method in the Fourier space

In this section, we first attain the solution formula. Then we obtain the decay properties of solution operators which is based on the pointwise estimate of solutions in the Fourier space.

2.1 Solution Formula

In this subsection, we try to obtain the solution formula for the problem (1.1) and (1.2). Assume that $G(x,t)$ and $H(x,t)$ are the solutions to the following problem:

$$\begin{cases} G_{tt} + (1 + \triangle^2)G - \triangle G_t = 0 \\ G(x,0) = \delta(x) \\ G_t(x,0) = 0 \end{cases} \tag{2.1}$$

$$\begin{cases} H_{tt} + (1 + \triangle^2)H - \triangle H_t = 0 \\ H(x,0) = 0 \\ H_t(x,0) = \delta(x). \end{cases} \tag{2.2}$$

Apply Fourier transform to (2.1) and (2.2), then we have formally that

$$\begin{aligned} & \hat{G}(\xi,t) \\ = \quad & Ce^{-\frac{|\xi|^2}{2}t}\Big(\frac{\sqrt{-4-3|\xi|^2}+|\xi|^2}{\sqrt{-4-3|\xi|^2}}e^{\frac{\sqrt{-4-3|\xi|^2}}{2}t} + \frac{\sqrt{-4-3|\xi|^2}-|\xi|^2}{\sqrt{-4-3|\xi|^2}}e^{-\frac{\sqrt{-4-3|\xi|^2}}{2}t}\Big), \end{aligned}$$

$$\hat{H}(\xi,t) = \frac{C}{\sqrt{-4-3|\xi|^2}}e^{-\frac{|\xi|^2}{2}t}\big(e^{\frac{\sqrt{-4-3|\xi|^2}}{2}t} - e^{-\frac{\sqrt{-4-3|\xi|^2}}{2}t}\big).$$

Here C is a constant determined by the initial data in (2.1) and (2.2).

In view of Duhamel principle, the solution to the problem (1.1) and (1.2) could be expressed as following:

$$u(t) = G(t) * u_0 + H(t) * u_1. \tag{2.3}$$

2.2 Decay Properties of Solution Operators

In this subsection, our aim is to obtain the decay estimates of the solution operators $G(t)*$ and $H(t)*$.

Proposition 2.1. *Let k be integers, $\varphi \in H^{s+1}(\mathbb{R}^n)\bigcap L^p(\mathbb{R}^n)$, $\psi \in H^{s-1}(\mathbb{R}^n)\bigcap L^p(\mathbb{R}^n)$, $1 \le p \le 2$, then the following estimates hold:*

$$
\begin{aligned}
&(1) \quad \|\partial_x^k G(t)*\varphi\|_{L^2} \le C(1+t)^{-\frac{k}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|\varphi\|_{L^p} + Ce^{-Ct}\|\partial_x^k\varphi\|_{L^2};\\
&(2) \quad \|\partial_x^k G_t(t)*\varphi\|_{L^2} \le C(1+t)^{-\frac{k}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|\varphi\|_{L^p} + Ce^{-Ct}\|\partial_x^{k+2}\varphi\|_{L^2};\\
&(3) \quad \|\partial_x^k H(t)*\psi\|_{L^2} \le C(1+t)^{-\frac{k}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|\psi\|_{L^p} + Ce^{-Ct}\|\partial_x^k\psi\|_{L^2};\\
&(4) \quad \|\partial_x^k H_t(t)*\psi\|_{L^2} \le C(1+t)^{-\frac{k}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|\psi\|_{L^p} + Ce^{-Ct}\|\partial_x^k\psi\|_{L^2};
\end{aligned}
$$

where $0 \le k \le s+1$ in (1) and $0 \le k \le s-1$ in (2)(3)(4).

To prove the proposition, the key point in to obtain the point-wise estimates of the fundamental solutions in the Fourier space. In fact this could be achieved by using the following point-wise estimate of solutions to the linear problem (1.1) and (1.2).

Lemma 2.1. *Assume u is the solution of* (1.1) *and* (1.2)*, then it satisfies the following pointwise estimate in the Fourier space:*

$$
|\hat{u}_t(\xi,t)|^2 + (1+|\xi|^4)|\hat{u}(\xi,t)|^2 \le Ce^{-C\rho(\xi)t}(|\hat{u}_1(\xi)|^2 + (1+|\xi|^4)|\hat{u}_0(\xi)|^2), \tag{2.4}
$$

here $\rho(\xi) = \frac{|\xi|^2}{1+|\xi|^2}$.

Proof. **Step 1:** Apply Fourier transform to (1.1), we have that

$$
\hat{u}_{tt} + (1+|\xi|^4)\hat{u} + |\xi|^2\hat{u}_t = 0. \tag{2.5}
$$

By multiplying (2.5) by $\bar{\hat{u}}_t$ and taking the real part, we have that

$$
\{|\hat{u}_t|^2 + (1+|\xi|^4)|\hat{u}|^2\}_t + 2|\xi|^2|\hat{u}_t|^2 = 0. \tag{2.6}
$$

Step 2:
By multiplying (2.5) by $\bar{\hat{u}}$ and taking the real part, we have that

$$
\{\frac{|\xi|^2}{2}|\hat{u}|^2 + Re\{\hat{u}_t\bar{\hat{u}}\}\}_t + (1+|\xi|^4)|\hat{u}|^2 - |\hat{u}_t|^2 = 0. \tag{2.7}
$$

Step 3:
By multiplying (2.7) by $|\xi|^2$ and plus (2.6), we have that

$$
\{|\hat{u}_t|^2 + (1+|\xi|^4)|\hat{u}|^2 + \frac{|\xi|^4}{2}|\hat{u}|^2 + |\xi|^2 Re\{\hat{u}_t\bar{\hat{u}}\}\}_t + |\xi|^2(1+|\xi|^4)|\hat{u}|^2 + |\xi|^2|\hat{u}_t|^2 = 0. \tag{2.8}
$$

Define $\rho(\xi) = \frac{|\xi|^2}{1+|\xi|^2}$, and denote

$$
E(\xi,t) = |\hat{u}_t|^2 + (1+|\xi|^4)|\hat{u}|^2 + \frac{|\xi|^4}{2}|\hat{u}|^2 + |\xi|^2 Re\{\hat{u}_t\bar{\hat{u}}\},
$$

$$
F(\xi,t) = |\xi|^2(1+|\xi|^4)|\hat{u}|^2 + |\xi|^2|\hat{u}_t|^2.
$$

There exists positive constant C such that the following relation holds:

$$C\rho(\xi)E(\xi,t)\le F(\xi,t). \tag{2.9}$$

Then (2.8) yields that:

$$\frac{\partial}{\partial t}E(\xi,t)+F(\xi,t)=0. \tag{2.10}$$

In view of (2.9) and (2.10), we have that:

$$E(\xi,t)\le e^{-C\rho(\xi)t}E(\xi,0). \tag{2.11}$$

We introduce Lyapunov functions:

$$E_0(\xi,t)=|\hat{u}_t|^2+(1+|\xi|^4)|\hat{u}|^2.$$

From the definitions of $E(\xi,t)$, we know that there exist positive constants $c_i (i=1,2,3)$ such that the following relation holds:

$$C_1E_0(\xi,t)\le E(\xi,t)\le C_2E_0(\xi,t). \tag{2.12}$$

By virtue of (2.11)and (2.12), we obtain the point-wise estimate of solutions to (1.1) and (1.2) in the Fourier space. □

As a simple corollary of Lemma 2.1, we have the following point-wise estimates of the fundamental solutions $G(x,t)$ and $H(x,t)$ in the Fourier space.

Lemma 2.2. *$G(x,t)$ and $H(x,t)$ satisfy*

$$\begin{aligned}
&(1)\quad |\hat{G}(\xi,t)|\le Ce^{-C\rho(\xi)t};\\
&(2)\quad |\hat{G}_t(\xi,t)|\le Ce^{-C\rho(\xi)t}(1+|\xi|^4)^{\frac{1}{2}};\\
&(3)\quad |\hat{H}(\xi,t)|\le Ce^{-C\rho(\xi)t}(1+|\xi|^4)^{-\frac{1}{2}};\\
&(4)\quad |\hat{H}_t(\xi,t)|\le Ce^{-C\rho(\xi)t};
\end{aligned}$$

where $\rho(\xi)=\frac{|\xi|^2}{1+|\xi|^2}$.

Proof. Putting (2.3) with $u_1=0$ in (2.4), it results that

$$|\hat{G}_t(\xi,t)|^2+(1+|\xi|^4)|\hat{G}(\xi,t)|^2\le Ce^{-C\rho(\xi)t}(1+|\xi|^4).$$

It yields (1) and (2) of Lemma 2.2.

Putting (2.3) with $u_0=0$ in (2.4), it results that

$$|\hat{H}_t(\xi,t)|^2+(1+|\xi|^4)|\hat{H}(\xi,t)|^2\le Ce^{-C\rho(\xi)t}.$$

It yields (3) and (4) of Lemma 2.2. □

Now we use Lemma 2.2 to prove Proposition 2.1.

Proof of Proposition 2.1. In view of (1) of Lemma 2.2, we have that

$$\begin{aligned}
&\|\partial_x^kG(t)*\varphi\|^2\\
\le\ & C\int_{R^n}|\xi|^{2k}e^{-C\rho(\xi)t}|\hat{\varphi}(\xi)|^2d\xi\\
\le\ & C\int_{\{\xi:|\xi|\le1\}}|\xi|^{2k}e^{-C|\xi|^2t}|\hat{\varphi}(\xi)|^2d\xi+C\int_{\{\xi:|\xi|\ge1\}}|\xi|^{2k}e^{-Ct}|\hat{\varphi}(\xi)|^2d\xi\\
=:\ & K_1+K_2.
\end{aligned}$$

Assume that p' satisfies $\frac{1}{p}+\frac{1}{p'}=1$, then

$$K_1 \le C(1+t)^{-k-n(\frac{1}{p}-\frac{1}{2})}\|\varphi\|^2_{L^p}, K_2 \le Ce^{-Ct}\|\partial^k_x\varphi\|^2_{L^2}, 0\le k\le s+1,$$

here $0\le k\le s+1$, thus (1) is proved.

Due to (2) of Lemma 2.2, it results that

$$\begin{aligned}
&\|\partial^k_x G_t(t)*\varphi\|^2\\
\le\ & C\int_{R^n}|\xi|^{2k}(1+|\xi|^4)e^{-C\rho(\xi)t}|\hat{\varphi}(\xi)|^2d\xi\\
\le\ & C\int_{\{\xi:|\xi|\le1\}}|\xi|^{2k}e^{-C|\xi|^2t}|\hat{\varphi}(\xi)|^2d\xi + C\int_{\{\xi:|\xi|\ge1\}}|\xi|^{2k+4}e^{-Ct}|\hat{\varphi}(\xi)|^2d\xi\\
\le\ & C(1+t)^{-k-n(\frac{1}{p}-\frac{1}{2})}\|\varphi\|^2_{L^p} + Ce^{-Ct}\|\partial^{k+2}_x\varphi\|^2_{L^2},
\end{aligned}$$

here $0\le k\le s-1$, thus (2) is proved.

It follows from (3) of Lemma 2.2 that

$$\begin{aligned}
&\|\partial^k_x H(t)*\psi\|^2\\
\le\ & C\int_{R^n}|\xi|^{2k}(1+|\xi|^4)^{-1}e^{-C\rho(\xi)t}|\hat{\psi}(\xi)|^2d\xi\\
\le\ & C\int_{\{\xi:|\xi|\le1\}}|\xi|^{2k}e^{-C|\xi|^2t}|\hat{\psi}(\xi)|^2d\xi + C\int_{\{\xi:|\xi|\ge1\}}|\xi|^{2k}e^{-Ct}|\hat{\psi}(\xi)|^2d\xi\\
\le\ & C(1+t)^{-k-n(\frac{1}{p}-\frac{1}{2})}\|\psi\|^2_{L^p} + Ce^{-Ct}\|\partial^{k}_x\psi\|^2_{L^2},
\end{aligned}$$

here $0\le k\le s-1$, thus (3) is proved.

(4) of Lemma 2.2 yields that

$$\begin{aligned}
&\|\partial^k_x H_t(t)*\psi\|^2\\
\le\ & C\int_{R^n}|\xi|^{2k}e^{-C\rho(\xi)t}|\hat{\psi}(\xi)|^2d\xi\\
\le\ & C(1+t)^{-k-n(\frac{1}{p}-\frac{1}{2})}\|\psi\|^2_{L^p} + Ce^{-Ct}\|\partial^{k}_x\psi\|^2_{L^2},
\end{aligned}$$

here $0\le k\le s-1$, thus (4) is proved. □

3 Decay Estimates for Linear Problem

In this section we study the decay estimates of solutions to the linear problem (1.1) and (1.2).

Theorem 3.1. *Let $s\ge1$ be an integer. Assume that $u_0\in H^{s+1}(\mathbb{R}^n)$ and $u_1\in H^{s-1}(\mathbb{R}^n)$, and put*

$$I_0 = \|u_0\|_{H^{s+1}} + \|u_1\|_{H^{s-1}}.$$

Then the solution u to the problem (1.1) and (1.2) given by (2.3) satisfies

$$u\in C^0([0,\infty)); H^{s+1}(R^n)\bigcap C^1([0,\infty); H^{s-1}(R^n))$$

and the following energy estimate:

$$\|u_t(t)\|^2_{H^{s-1}} + \|u(t)\|^2_{H^{s+1}} + \int_0^t \|\partial^2_x u_t(\tau)\|^2_{H^{s-3}} + \|\partial^2_x u(\tau)\|^2_{H^{s-1}}d\tau \le CI_0^2.$$

Proof. We have obtained the solution u of (1.1) and (1.2)given by (2.3) and proved that it satisfies the point-wise estimate (2.4) in the Fourier space. From (2.9) and (2.10) we have that

$$\frac{\partial}{\partial t}E(\xi,t)+C\rho(\xi)E(\xi,t)\le 0.$$

Integrate the inequality with respect to t and appeal to (2.12), then we obtain

$$E_0(\xi,t)+\int_0^t \rho(\xi)E_0(\xi,\tau)d\tau \le CE_0(\xi,t). \tag{3.1}$$

Multiply (3.1) by $(1+|\xi|^2)^{s-1}$ and integrate the resulting inequality with respect to $\xi\in\mathbb{R}^n$, then we have that

$$\|u_t(t)\|^2_{H^{s-1}}+\|u(t)\|^2_{H^{s+1}}+\int_0^t \|\partial_x^2 u_t(\tau)\|^2_{H^{s-3}}+\|\partial_x^2 u(\tau)\|^2_{H^{s-1}}d\tau\le CI_0^2. \tag{3.2}$$

(3.2) shows that we complete the proof of Theorem 3.1. □

By using Proposition 2.1 with $p=2$, we obtain the following decay estimate of u given by (2.3), if initial data $u_0\in H^{s+1}(\mathbb{R}^n)$ and $u_1\in H^{s-1}(\mathbb{R}^n)$.

Theorem 3.2. *Under the same assumption as in Theorem 3.1, then u given by* (2.3) *satisfies the decay estimate:*

$$\|\partial_x^k u(t)\|_{H^{s+1-k}}\le CI_0(1+k)^{-\frac{k}{2}}, 0\le k\le s+1. \tag{3.3}$$

Proof. Let $k\ge 0$, $m\ge 0$ be integers, In view of (2.3), by using (1) and (3) of Proposition 2.1 with $p=2$, we have that

$$\begin{aligned}
\|\partial_x^{k+m}u(t)\| &\le \|\partial_x^{k+m}G(t)*u_0\|_{L^2}+\|\partial_x^{k+m}H(t)*u_1\|_{L^2}\\
&\le C(1+t)^{-\frac{k+m}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|u_0\|_{L^2}+Ce^{-Ct}\|\partial_x^{k+m}u_0\|_{L^2}\\
&\quad +C(1+t)^{-\frac{k+m}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|u_1\|_{L^2}+Ce^{-Ct}\|\partial_x^{k+m}u_1\|_{L^2}\\
&\le C(1+t)^{-\frac{k}{2}}\|u_0\|_{L^2}+C(1+t)^{-\frac{k}{2}}\|u_1\|_{L^2}\\
&\quad +Ce^{-Ct}\|\partial_x^{k+m}u_0\|_{L^2}+Ce^{-Ct}\|\partial_x^{k+m}u_1\|_{L^2}\\
&\le Ce^{-Ct}\|\partial_x^{k+m}u_1\|_{L^2}+Ce^{-Ct}\|\partial_x^{k+m}u_0\|_{L^2}\\
&\quad +C(1+t)^{-\frac{k}{2}}\|(u_0,u_1)\|_{L^2},
\end{aligned}$$

here $k\ge 0$, $k+m\le s-1$. Then we obtain that

$$\|\partial_x^{k+m}u(t)\|_{L^2}\le CI_0(1+t)^{-\frac{k}{2}},$$

here $0\le m\le s+1-k$. Take sum with $0\le m\le s-1-k$, we obtain (3.3). Thus Theorem 3.2 is proved. □

Remark 3.1. Under the same assumption as in Theorem 3.1, u given by (2.3) also satisfies the following decay estimate:

$$\|\partial_x^k u_t(t)\|_{H^{s-1-k}}\le CI_0(1+k)^{-\frac{k}{2}}, 0\le k\le s-1.$$

If we assume the initial data belong to $L^1(\mathbb{R})$, then we have the following sharp decay estimates.

Theorem 3.3. *Let $s \geq 1$ be an integer. Assume that $u_0 \in H^{s+1}(\mathbb{R}^n) \bigcap L^1(\mathbb{R}^n)$ and $u_1 \in H^{s-1}(\mathbb{R}^n) \bigcap L^1(\mathbb{R}^n)$, and put*

$$I_1 = \|u_0\|_{H^{s+1}} + \|u_1\|_{H^{s-1}} + \|(u_0, u_1)\|_{L^1}.$$

Then the solution u to (1.1) *and* (1.2) *given by* (2.3) *satisfies the following decay estimates:*

$$\|\partial_x^k u(t)\|_{H^{s+1-k}} \leq CI_1(1+t)^{-\frac{k}{2}-\frac{n}{4}}. \tag{3.4}$$

Proof. Let $k \geq 0$, $m \geq 0$ be integers, In view of (2.3), by using (1) and (3) of Proposition 2.1 with $p = 1$, we have that

$$\begin{aligned}
\|\partial_x^{k+m} u(t)\|_{L^2} &\leq \|\partial_x^{k+m} G(t) * u_0\|_{L^2} + \|\partial_x^{k+m} H(t) * u_1\|_{L^2} \\
&\leq Ce^{-Ct}\|\partial_x^{k+m} u_0\|_{L^2} + C(1+t)^{-\frac{k+m}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|u_0\|_{L^1} \\
&\quad + Ce^{-Ct}\|\partial_x^{k+m} u_1\|_{L^2} + C(1+t)^{-\frac{k+m}{2}-\frac{n}{2}(\frac{1}{p}-\frac{1}{2})}\|u_1\|_{L^1} \\
&\leq Ce^{-Ct}\|\partial_x^{k+m} u_0\|_{L^2} + Ce^{-Ct}\|\partial_x^{k+m} u_1\|_{L^2} \\
&\quad + C(1+t)^{-\frac{k}{2}-\frac{n}{4}}\|(u_0, u_1)\|_{L^1},
\end{aligned}$$

with $k \geq 0$, $k + m \leq s - 1$. Then we have that

$$\|\partial_x^{k+m} u(t)\|_{L^2} \leq CI_1(1+t)^{-\frac{k}{2}-\frac{n}{4}},$$

here $0 \leq m \leq s - 1 - k$. By taking sum with $0 \leq m \leq s - 1 - k$ we obtain (3.4). Thus Theorem 3.3 is proved. □

4 Conclusions

a In the section (3), the decay estimates and sharp decay estimates of solutions to (1.1) and (1.2) obtained in Theorem 3.2 and Theorem 3.3 show that the dissipative term $-\Delta u_t$ has effect on the decay of solutions.

b In the section (3), the decay estimates and sharp decay estimates of solutions to (1.1) and (1.2) obtained in Theorem 3.2 and Theorem 3.3 show that the dissipative structure is not of regularity-loss type.

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Competing Interests

Authors have declared that no competing interests exist.

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