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The Development of Preschool Students' Algebraic Thinking Skills: A Case Study Approach

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Authors' contributions

This study was carriedout in collaboration between two authors. Both authors designed thestudy, author TÖ got necessary permissions from necessary institutions. Authors CI and TÖ carriedout all data gathering process and performed necessary data analysis. Authors CI and TÖ wrote the first draft of the manuscript together. Both authors managed the literature searches and author CI edited the manuscript. Both authors read and approved the final manuscript.

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ABSTRACT

Basic mathematical skills which are significant for furthere ducation levels and also for being successful in real life are gained during early childhood education period. This study was done in Agriprovince with four preschool students in repeating, growing and relational patterns to analyse their algebraic thinking skills. In this case study approach, criterion sampling method was used. With respect to looking for pattern, identifying and defining patterns and generalising patterns steps of algebraic thinking skill, findings indicated that participants, who didn't have experience in growing patterns in their previous lessons, were more successful in the sepatterns than repeating patterns they had experienced.

Keywords: Early childhood education; preschool students; algebraic thinking skill; pattern types.

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1. INTRODUCTION

Basics of our learning are founded generally in early childhood period and education gained in this period affects further life of an individual. This period, at the same time, has a critical significance in individual's life for gaining of basic knowledge, skills and habits that in a sense prepare individual to real life. When thought in the context of mathematics, basic mathematical knowledge is also founded in this period. A good early childhood education, as Clements [1] mentioned, is beyond counting and addition practices, instead it is deeper and more comprehensive. In this context, it is significant that children attending early childhood education go through rich mathematical experiences, so as a result they are expected to be successful in mathematics both during their further primary education process and in their real life [2-4].

Early childhood education, as stated before, is a period of basic developmental changes gained and moreover, mathematical concepts are formed meanwhile [1]. Therefore, providing rich mathematical experiences will help children solve real life problems as well as making them successful in other disciplines. In Turkey, early childhood education curriculum was updated in 2013. In this curriculum, children are supposed to gain classification, matching, comparing, sorting, number concept, operations, spatial sense and geometry, measurement and graphics in accordance with National Council of Teachers of Mathematics (NCTM) content standards. For children to have experience in these skills, various activities done in lessons by their teachers are rather significant. Using patterns is also an activity they generally apply in their lessons.

In the context of patterns, while Steen [5] defined mathematics as a science of patterns, Van de Walle [6] stated that mathematics deal with the rule of patterns and discover, interpret and use them. Hence, pattern concept reminds an order and a rule. These features could be seen in different definitions in related literature [see, e.g., 7,8]. Based on these definitions, patterns could be expressed as a systematic combination of geometric shapes, sounds, symbols or actions and it is a numeric and spatial regularity. In related literature, there are *repeating, growing* and *relational* patterns [9]. Repeating pattern is composed of repetition of basic segment and it has a cyclical structure of this smallest segment. In growing patterns, basic segment is used as a block for generating a bigger group and each new step is related with the previous step [10]. Relational patterns, as concerns, are done based on the relation with associated two pattern sets [11].

In development of recognizing mathematical relations, generalizing and comprehending mathematical order, skills like identifying, continuing and constructing patterns have significant place [9]. Identifying patterns is a significant process, and besides, it is a basic element of children's mathematical development. Yet, identifying patterns is asserted as propulsive force for kids [12]. Therefore, it is significant for preschool children encounter with activities and necessitates them to identify, construct number and shape patterns, and generalize these situations to real life issues.

According to Tanisli and Ozdas [13], patterns are building blocks of generalization and generalization is building block of algebra. So, it should be insisted on patterns for making generalizations. When algebra is considered specifically, it is a language and this language has five basic elements; formula, unknown, variable, relation and generalization [14]. Briefly algebra could be defined as the generalization of arithmetic. In addition, NCTM [15] defined algebra as a basic segment of mathematics curriculum from preschool to higher education in school mathematics principle and standards document. On the other hand, algebra is one of the learning areas that students found compelling and students had difficulties. Many people perceive algebra as a collection of rules including simplification of algebraic equations, solving equalities, using symbols and accordingly, they hate algebra. According to Kaput [16], main reason behind this situation is the fact that in schools algebra is taught as a range of rules and independent of other topics in mathematics and it is taught as there is not a relation between it and the real life. For overcoming this problem, mathematics educators have been studying about with which teaching methods algebra should be taught. An insisted one among all is using patterns before teaching algebra. In Turkey, in primary mathematics curriculum it is stated that patterns should be shown in different forms and specifically showing them symbolically helps them form the basic terms of algebra in the beginning of learning algebra [17].

Skills like identifying, continuing patterns, finding out the rule for getting the next step, and stating

a rule both verbally and symbolically direct students to algebraic thinking [18]. To NCTM [19], algebraic thinking necessitates understanding functions, representing and analysing mathematical structures and situations in different ways and also necessitates for representing and understanding quantitative relations using mathematical models and analysing various situations in real life. In addition, algebraic thinking includes the prediction of demonstrating, doing, generalizing patterns and regularities [20]. Steele [21], additionally, indicated algebraic thinking includes three steps like; looking for a pattern, identifying and defining patterns and lastly generalizing patterns. While looking for a pattern step deal with revealing knowledge from a problem situation, in identifying and defining step knowledge is represented mathematically by with words, shapes, tables, graphics and equations. Lastly, generalizing pattern step includes interpreting and applying mathematical findings with issues like finding the unknown, examining the assumption and defining as a functional relation [22].

According to NCTM [19], in early ages children's algebraic concepts can be improved, working with numbers and patterns would help providing a basis to children's algebraic thinking and problem solving skills. Generalization skill is also significant for development of algebraic thinking skill. Generalization skill is stated as an essence of mathematics [23] and this situation is explained as one of the aims of the mathematics as in NCTM standards [19]. It is significant to improve algebraic thinking in early childhood period. This period should be enriched with activities including different types of patterns. In the content of current preschool curriculum there are activities about patterns, hence, it could be stated that this curriculum aims to improve algebraic thinking.

When researches done are analysed, there are studies about algebraic thinking process, generalization skill, and how these skills are related based on the content of algebra learning area or patterns only with primary and high school with students, teachers or teacher candidates. Bas, Erbas and Cetinkaya [24] tried to discover high school mathematics teachers' knowledge and thoughts about 9^{th} grade students' algebraic thinking structures. At the end of this study, they found out teachers' expectations about their students' algebraic thinking structures and the students' performance in generalization activity were very different significantly. Akkan and Cakiroglu [25] compared 6^{th} and 8^{th} grade students' generalization strategies in linear and quadratic based patterns and they found out that when the grades got higher students' generalization strategies were diversified and the ability to get correct generalization results were increased. In a pattern based and non-pattern based algebra teaching, quasi-experimental design was realized with 7^{th} grade students [18]. At the end of the study, they found a meaningful difference based on Conceptual Algebra Test between two groups. Besides, they found that pattern based algebra teaching had positive effects on algebraic thinking skills and increased positive attitude towards mathematics. 5^{th} grade students' generalization strategies was analysed in Tanisli and Ozdas [13] study. As a result, they reached that in generalizations of repeating and growing patterns, they internalized visual and algebraic approaches. Moreover, they investigated that students used pattern generalization strategies frequently in close and distant generalizations. In a study realized with middle school level, Bagdat [26] investigated 8^{th} grade students' algebraic thinking skills according to solo taxonomy. As a result, participating students had problems in usage of symbols and algebraic relations and besides, students, who had higher academic success, had higher algebraic thinking skills. In studies with middle and higher education level, they generally emphasized the importance of patterns in algebra teaching.

In studies done with teacher candidates, for instance, Tanisli and Kose [27] investigated candidate classroom teachers' cognitive structures in their generalization process. Researchers found out that the experiment they applied improved teacher candidates' generalization skill in patterns to algebra. Celik [28], on the other hand, tried to characterize mathematics teacher candidates' algebraic thinking skills according to solo taxonomy. Researcher discovered that most of the teacher candidates were not good at using symbols and algebraic relations, or making use of multiple representations and formulating generalizations, and the reason behind this was thought to be not being able to integrate consistently the knowledge and the skill they had.

In some studies about early education period, Fox [29] investigated how teachers' got involved to process and how this affected children's participation to mathematical patterning before

compulsory education. As a result of this study, these activities provided children richer environment and teachers' role during this process is found to be significant. Another study was realized by Papic and Mulligan [7], they searched the development of preschool students' constructing pattern skill in matched schools and they found that students, who participated constructing pattern program, were more successful.

Although in the literature there are studies about algebraic thinking skill and how patterns affect it and in most of the studies they suggest including this skill to early childhood education, specifically in Turkey it is hard to encounter these studies. When this situation is considered, the significance of this study could be seen. In the light of all these studies, main aim of the study to analyse preschool students' algebraic thinking process done based on different pattern types. For this purpose, research question is preschool students' (60-72 months) algebraic thinking process realized based on different pattern types (Repeating, growing, and relational patterns)?

2. METHODS

2.1 Design

This study is a case study, as Yin [30] mentioned it attempts to study a current phenomenon in its own real context and its limits between phenomenon and context are not clearly evident; besides, in this method there is more than one evidence or data source. In this study, it was aimed to determine 60 to 72 months old preschool students' development of algebraic thinking skills with regard to pattern types. Appropriate to case study method, data were gathered from a preschool in Agri Province.

2.2 Participants

This study was done with four preschool students selected according to criterion sampling method during autumn term in 2013-2014 academic year. Two of the participants were female and other two were male. In selection of participants, students' being talkative, expressing their thoughts confidently and being successful in previous mathematical activities were the criteria accepted. According to participating students' teacher's expressions and opinions, these participants had these criteria. Besides, participants had been getting preschool education for two and half years and had participated activities about repeating patterns beforehand.

2.3 Procedureand Instrumentation

When necessary permissions were taken from Agri Province National Education Directory, during three weeks period in different times each part of the data gathering tool was applied to participants one by one. And each part of the tool was applied in different times. Before implementation of tool, participants' teacher introduced the participants the researchers and explained them how the procedure would be done. Implementation of tool was conducted at school where school principal provided. During implementation process, school directory didn't give permission to video recording, so voice recording tool was used. Therefore, participants solving process and their expressions were recorded with it. In addition, researchers took in-depth notes. Data gathering done in each week took about 10-20 minutes for each student.

For data gathering, a tool which had three parts appropriate to three pattern types was designed. In designing of the tool, researchers benefitted from related literature and the opinions of two experts who had studies in various areas in mathematics education. In addition to these, suggestions of a preschool teacher working in Agri Province were taken into consideration. In accordance with two experts' opinions and a preschool teachers' suggestions data gathering tool had its final form.

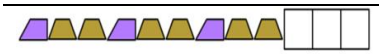
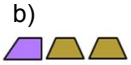
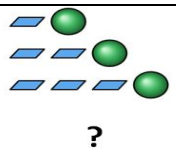
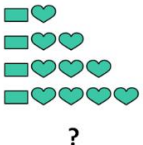
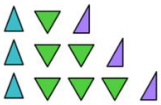
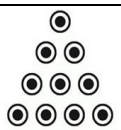
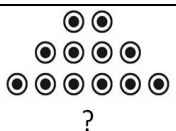
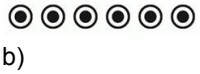
Data gathering tool had three parts; it had totally 12 questions including repeating, growing and relational patterns, respectively. Moreover, for each question there were three options presented and participants were asked to choose one. Yet, while implementation of data gathering tool some participating students preferred drawing their correct answer and then, they chose an option presented to them. Drawing has a significant place in early childhood education, so this might be the reason behind this situation. Participants didn't only indicate correct answer they thought, but also asked to explain why and how they did to the researchers. In the following Table 1 the questions asked about different pattern types and given options are presented.

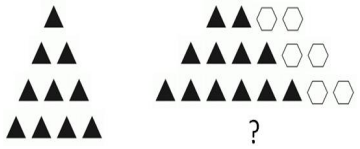
2.4 Data Analysis

Before analysing data, participating students were coded with pseudo names; Ecrin, Arda,

Yusuf and Zeynep. Answers of participants to each question asked appropriate to three pattern types were categorized as correct, incorrect and no answer. In addition, participants' expressions recorded were determined. These expressions were analysed for how they supported or didn't support the participants' answers.

Table 1. The instrument

Pattern types	Questions	Options given
1) **Repeating patterns**	2 1 3 2 1 3 2 1 3 ☐☐☐	a) 1 3 2 b) 2 1 3 c) 1 2 1
		a)  b)  c) 
		a)  b)  c) 
		a)  b)  c) 
2) **Growing patterns**	 ?	a)  b)  c) 
	 ?	a)  b)  c) 
	 ?	a)  b)  c) 
	 ?	a) 1 1 b) 1 1 1 1 1 1 c) 1 1 1 1
3) **Relational patterns**	  ?	a)  b)  c) 

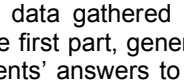
Pattern types	Questions	Options given
		a)  b)  c) 
		a)  b)  c) 
		a)  b)  c) 

3. RESULTS

In this section, data gathered are presented in two parts. In the first part, general findings about preschool students' answers to the all questions are given. In the second part, participants' finding pattern segment process in repeating, growing, and relational patterns are explained in detail.

3.1 General findings

Participating students' answers are categorized as correct, incorrect or no answer; as a result, these categorizations are presented in the following Table 2.

As seen in Table 2, participating students' total numbers of correct answers to repeating, growing, and relational patterns are 12, 13, and 7, respectively. When participants' answers are analysed one by one according to different pattern types; in all '  , 213,' repeating patterns, in '  , 1' growing patterns, and lastly in '  ' relational pattern, students had more correct answers with respect to other pattern questions. When participants' correct answers to pattern questions are analysed in general; all participants gave correct answers to '' and '1' growing patterns. Least correct answers were given by participants to '  ' and '  ' relational pattern questions. In data gathered, participants had more correct answers in growing patterns (f=13) although they had experience in repeating patterns in their previous lessons. On the other hand, participants has least correct answer in relational patterns (f=7). Students' pattern solving process in different types is presented in the following section.

3.2 Findings Related with Pattern Types in Detail

3.2.1 Findings related with repeating patterns

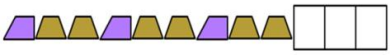
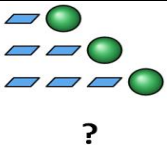
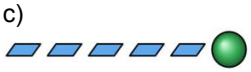
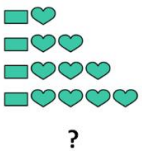
There were four questions asked preschool students regarding repeating patterns having 1-1, 2-1, 1-2 and 1-1-1- rules. In the following of this part, participants' answering processes are explained.

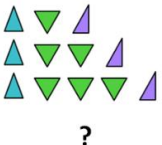
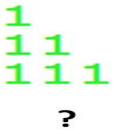
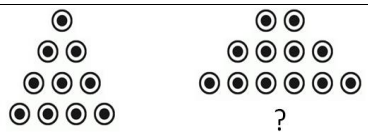
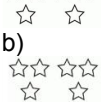
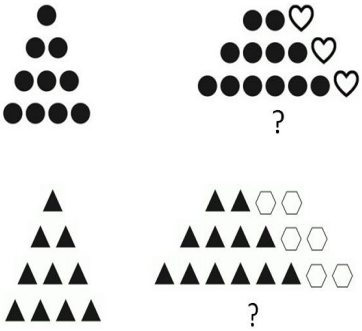
First of all, appropriate to 1-1 pattern rule '  ' question was asked. All of the students except Yusuf answered this question correctly. Before determining correct option for pattern segment, Ecrin and Arda counted all the colours loudly. Both drew pattern segment to given empty space firstly (Fig. 1) and then, they chose

an option which was as same as what they drew. Zeynep, on the other hand, drew pattern segment she thought to be correct as soon as she saw the question. Like other students, Yusuf showed the option he thought to be correct as soon as he saw and he added that this question was so easy. However, his answer was incorrect.

Fig. 1. Ecrin's answer as a drawing

Table 2. Thedistribution of preschoolstudents' answerstotheinstrument

Pattern types	Questions	Options given
1) Repeating patterns	2 1 3 2 1 3 2 1 3 ☐☐☐	a) 1 3 2 b) 2 1 3 c) 1 2 1
		a)  b)  c) 
		a)  b)  c) 
		a)  b)  c) 
2) Growing patterns		a)  b)  c) 
		a)  b)  c) 

Pattern types	Questions	Options given
		a)  b)  c) 
		a) b)  c) 
3) Relational patterns		a)  b)  c) 
		a)  b)  c) 
		a)  b)  c) 
		a)  b)  c) 

In ‘  ‘ question appropriate to 2-1 rule, Ecrin instead of choosing correct answer from given options, firstly she wanted to draw correct answer she thought to be correct in the given empty spaces. Then, she tried to choose the same option as same as what she drew. After she solved this, she added that this question was so easy to solve. Arda also did the same procedures as Ecrin went through (Fig. 2). Zeynep, on the other hand, read all the pattern segments loudly at first. Then, she said the answer was ‘  ,’ but this was incorrect. In Zeynep’s choice of this incorrect answer, Zeynep might still think as in the first question asked appropriate to 1-1 pattern rule. Consequently, she interpreted this question was as same as the first question. Yusuf told pattern segment in the current one by showing shapes and he showed the correct answer from the given options.

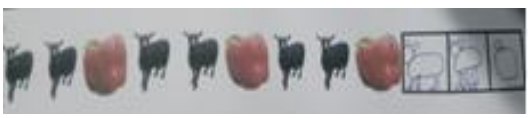


Fig. 2. Arda’s answer as a drawing

Ecrin tried to read the colours of shapes firstly in ‘  ‘question asked appropriate to 1-2 pattern rule as she did in other pattern questions asked previously. Lastly, she drew correct pattern segment in the empty spaces by saying their colours loudly (Fig. 3.). Arda also read all colours loudly; then, he said which shapes would be in the empty spaces by including colours. However, he indicated incorrect one from the options as his correct answer, after that he noticed his fault. Then, he showed the correct answer. Zeynep had a difficulty in this question and she showed ‘  ’ pattern segment as her correct answer. In this question, Zeynep had the same fault in this question like in ‘  ’ question; that is to say, she reached incorrect answer as a result. It is thought that she generalized the rule of ‘  ’ pattern to this question again; 1-1 pattern rule. Yusuf, finally, said the pattern segment loudly with adding their colours as Ecrin and Arda did in this question. Then, he stated pattern segment correctly and added ‘I can do all questions correct’ with self-confident manner.

Fig. 3. Ecrin’s answer as a drawing

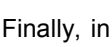
Finally, in ‘  ’ pattern question appropriate to 1-1-1 pattern rule, Ecrin as she did in the first question, drew what she thought to be correct in the empty spaces. Then, she chose the correct option as same as she drew (Fig. 5). When Ecrin was asked about the reason of her choice of ‘  ’ pattern segment, she answered that this was as same as what she drew and she tried to prove this by showing the researchers. Arda and Yusuf firstly analysed the pattern given. When they drew correct answer in the empty spaces, they also showed correct option. After showing correct answer, both Arda and Yusuf added that this question was easy. Besides, Arda mentioned that he wanted to solve more questions. Zeynep like other participating students read the numbers and without focusing on the pattern rule she said the next pattern segment would be four. Then, she recognized that there was not any option like she said. She drew her correct answer and she chose an option uncertainly (Fig. 4), and her answer was incorrect.

Fig. 4. Zeynep’s answer as a drawing

Fig. 5. Ecrin’s answer as a drawing

When the findings about repeating patterns are analysed, generally Arda, Ecrin and in some questions Yusuf and Zeynep tried to find the pattern rule, first of all, by reading all patterns loudly. Participating students although they were not required to draw, wanted to draw pattern segment in the given empty spaces. Then, they tried to find the option as same as what they drew. According to participants’ teacher, they had done repeating patterns practices beforehand. However, only Ecrin and Arda answered all these pattern types correctly. Zeynep, on the other hand, could reach only one correct answer which was asked appropriate to 1-1 pattern rule.

About repeating patterns, participants while solving the questions looked for a pattern by reading the patterns loudly. By this way, they were trying to reveal their knowledge. In the second step ‘identifying and defining patterns,’ almost all participants in each question tried to draw their answer in the given spaces or controlled by reading the shapes they drew. In the generalization step, almost all participants again chose an option from the given ones.

3.2.2 Findings related with growing pattern

In this first question of growing patterns ‘  ,’ while the number of green circle is constant, the number of blue parallelogram is increasing one by one in each step. In this question fourth step was asked to participants. While Arda was answering this question, he noticed the increase in each step and added this. Later on he recognized that the number of green circles was constant in each step. He counted all blue parallelograms in each step one by one; he added that there would be four blue parallelograms in the fourth step. Then, he counted blue shapes in all options one by one and chose correct answer. When Arda was asked why he thought like that, he put his choice

on the question mark and he again controlled. Moreover, he stated 'it fits the empty space.' He added that he thought the question was easy. Ecrin, as she did in repeating patterns, read and counted all the shapes in each step loudly. Then, she mentioned about the increase, however, she added that the number of green circles in each step stayed the same. She stated that in the third step there were three parallelograms, so she added there should be four in the fourth step. She analysed each option given and she controlled all options by putting them on the question mark one by one. Yusuf before answering this question looked over it and reached out an option. He counted the parallelograms in this option and he added that there should be four of them. He stated that the option he had in his hand was incorrect. Then, he determined the number of parallelograms in each option and he added the number of green circle was always the same. Lastly, Zeynep before answering this question, like the other participants counted the parallelograms and circles in each step one by one. She mentioned the researchers that the number of green circle was constant. However, while trying to solve correct answer he put all the options on the question mark and controlled. Although she tried to solve, she chose an incorrect answer and indicated this to the researchers. In this pattern question, all the participating students noticed the constant number of green circle and increased number of blue parallelogram in each step. At the same time, participants as distinct from repeating patterns put all the options given on the question mark and controlled them.

In '  ' question, the number of rectangle is constant and the number of heart is increasing one by one in each step. While answering this question, Arda mentioned the increase in general. Then, he mentioned that the number of rectangle was constant but the number of heart was increasing in each step. He counted all the options given and he added that there would be five hearts in the fifth step. He showed an option by putting it under the question. Ecrin before answering this question counted all the shapes in each step and tried them by putting under the pattern. Then, she analysed all options and showed one she thought to be correct. Yusuf and Zeynep counted all the shapes in each step like Ecrin did. Then, they counted options and showed one to the researchers. When Yusuf was asked why he chose this option, he mentioned that there would be five after four. Zeynep answered the same question, but she had a problem to express her and answered shortly as 'just like this.' All the participating students were able to answer this pattern question correctly.

In '  ' growing pattern question, in each step number one is increasing one by one and the fourth step is asked. In this pattern all the participants could find correct answer. While Arda was solving this question, he counted all ones and added there should be four in the fourth step. He counted all options, put them on the question mark and controlled one by one. Ecrin also did the same things as Arda did, but she chose an incorrect one at first. When she controlled the number of ones, she noticed that it was incorrect and found out correct answer. In this question Yusuf first of all counted all the ones totally instead of counting the ones in each step one by one. Then, he noticed that he should analyse the steps one by one and stated that there should be four ones in the fourth step. When Yusuf was asked why he chose this option, he said shortly that there should be four after three. Zeynep answered this question by following the same steps as other did and found correct answer. In this pattern there was only one variable, this might the reason why all the participants answered this correctly.

Finally, in '  ' pattern question, both the numbers of blue and purple triangles at the beginning and at the end are constant, but the number of green triangle in the middle is increasing one by one in each step. While Arda was answering this question, he said the researchers that the numbers of blue and purple triangles were not increasing but the number of green one was increasing in each step. Then, he counted all the triangles in the given options; that is to say, all the triangles without allowing the constant ones, he showed an incorrect option to the researchers. Ecrin followed the same process as Arda did. She said loudly the colours and the numbers of the shapes one by one. Without counting the numbers in the given options, she looked over the options and controlled them. She chose an incorrect option. Yusuf and Zeynep, who answered correctly to this pattern question, counted the numbers of triangles including their colours. In this step, they noticed that some colours stayed the same and told this to the researchers. At the end, they controlled all the options by putting them on the question mark and found correct answer. When they were asked about how they could do it, Yusuf mentioned that only the number of green

triangles was increasing. He added that he could solve with this piece of information. Zeynep answered the same question as she put correct one on the question mark, controlled it and found the correct answer. In this pattern question, all participants noticed that the number of green triangles was increasing but the numbers of others were constant. Besides, it is determined that all participants focused on colours and counted the numbers of them.

According to participants' teacher's expressions, students had not participated growing patterns activities before. But when participating students' general correct answers are analysed, students had more correct answers in this pattern type. Besides, as distinct from repeating patterns, in growing patterns students counted all the shapes and options, they tried to control the options they chose by putting them on the question mark. When these findings are considered according to the steps of algebraic thinking skill, students' reading patterns loudly or telling the constants of growing segments are issues related with the first step 'looking for a pattern.' About identifying and defining patterns step, participants told the researchers what could be in the next step and they tried to control their answer by putting their answer under the question. In the third step 'generalization', they showed their answer after their control processes.

3.2.3 Findings related with relational patterns

In this type of pattern, there are four questions asked appropriate to 'y=2x+1,' 'y=2x,' 'y=2x+2' and lastly 'y=3x' expressions. According to participating students' teacher, in the school students did not have experience in relational patterns too.

In the first question related with relational patterns, '● ; ●●♡' pattern question appropriate to 'y=2x+1' expression was given. In this question, Arda without counting steps and patterns only put the options on the question mark and controlled. Finally, he chose an incorrect one. Ecrin counted all the shapes in each step before answering the question. Then, she determined the numbers of shapes in the options and picked up an incorrect one. Yusuf immediately controlled the choices and put them on the question mark. He noticed the number of hearts was constant but chose an incorrect answer. Zeynep who found correct pattern segment counted all steps in each pattern set. She noticed that there was only one heart in each step. First of all, she chose an incorrect one. However, when she noticed that the number of heart was not appropriate to what she said before, she showed correct answer.

'◉ ; ◉◉' pattern question appropriate to 'y=2x' expression was asked. Arda in this pattern question counted all the shapes in each step. Then, he showed an option which had six shapes. When Arda was asked about his choice, he showed that in the second pattern set there were six shapes. Ecrin followed the same process as Arda did and found an incorrect answer. Yusuf in this question counted the shapes two by two. After analysing the options given, he showed correct answer. When Yusuf was asked why he chose that option, he indicated that eight is close to six as his reason of choice. Lastly, Zeynep controlled all the options by putting them on the question mark. Then, she showed an option and remarked that the number of shapes was not more than the ones in question, so she chose that option. Although, her reason of choice didn't fit the reality, she chose the correct answer. It was thought that Yusuf's counting two by two was effective to find correct answer. Zeynep's finding of correct answer was thought to be the result of controlling process.

'▲ ; ▲▲⬡⬡' question appropriate to 'y=2x+2' expression was given. Before answering, Arda counted all the shapes in the pattern sets. Then, he controlled all the options by putting on the question mark. When Arda thought that he could not come up with a solution, he did not give an answer. Ecrin, Zeynep and Yusuf noticed that in the second set there were two hexagons at the end. Therefore, they counted only the triangles, they found correct answer. Yusuf stated that he controlled all the options, but in two of the options the number of hexagons was less than the questions and in another option the place of hexagons was in the incorrect place. Therefore, the options given and noticing the constant number of hexagons helped Ecrin, Yusuf and Zeynep to find correct answer.

In the last pattern, '☆ ; ☆☆' pattern question appropriate to 'y=3x' expression was given. Ecrin, who found correct answer, counted the stars as sets of three stars. She controlled all the options by putting them on the question mark

and found correct answer. Arda, Yusuf and Zeynep counted all the stars in question and options given before answering the question. Then, they controlled each option by putting on the question mark. Then, they showed an incorrect answer. The reason behind how Ecrin found the correct answer is thought to be that Ecrin noticed the way of presentation in the second part of the pattern.

When general findings in relational patterns are analysed, it is thought that students' controlling of options one by one were affective in their solving process like in the growing patterns. Zeynep was more successful in this type of patterns than repeating patterns. The reason behind her success may be the result of how the patterns are presented or the options given. Arda was successful in repeating and growing patterns but he could not reach one correct answer in this pattern type which is also thought to be a significant result. The reason behind this finding is thought to be that he did not have an experience this type of patterns before. Participants generally had difficulties in this pattern type.

About first step of algebraic thinking they tried to look over the pattern and tried to reveal their knowledge. However, they generally could not reach pattern rule. So, in further steps they generally could not reach correct answers. Some participants found out correct options and their reasons lied on different assumptions as mentioned before.

4. DISCUSSION AND CONCLUSİON

In Turkish preschool curriculum, students are expected to construct patterns with objects and to find the rule of patterns which has at most three variables [31]. In early childhood education period repeating patterns are used in activities. In this study, while participant preschool students were answering questions about repeating, growing and relational pattern types like in other many studies seen in literature, they primarily tried to analyse structural relations of given shapes in patterns [25,18,7,13]. During data gathering process, while some students repeated loudly, some determined the numbers of shapes or the others tried to find correct answer by trial and error method. In a sense, as Tanisli and Ozdas [13] indicated in their studies, they tried to reach pattern segments in questions by internalizing both quantitative and visual approaches.

Some participants read patterns in questions loudly; some participants looked over patterns and options. When participants saw the patterns, in a sense they were tended to look for a pattern, to identify and define patterns in this situation. Then, while drawing the pattern segments in the empty spaces in questions or while showing or controlling the option they thought to be correct they are in generalization step. While they were going through these steps, participating students in each pattern type went through the algebraic thinking processes as Steele [21] defined.

Although, students were asked to choose correct option, some of the students drew the pattern segment as their correct answer in empty spaces and then, choose the appropriate answer from the given options. Children chose to draw picture, in a sense, this could be explained with Artut's [32] expression 'drawing is a way to express oneself uniquely and simply with showing natural images romantically and intellectually'. In this study, some students chose drawing firstly, it is thought that drawing is a way that children chose to communicate.

In growing and relational patterns although participants counted the shapes or numbers, they wanted to control the options. In a sense, with a visual approach, they wanted to control the option if it fitted or not. Participants went through Steele's definition of algebraic thinking process steps; looking for a pattern, identifying and defining patterns in growing and relational patterns. However, they were not adequate in specifically generalizing of relational patterns.

One another significant finding was; although, Zeynep had experience in repeating patterns, she could not be successful in this type like other students. However, she was as successful as others in growing and relational patterns. In fact, she was more successful than others in relational patterns. Zeynep's success in '●; ●●♡ and ▲; ▲▲⬡⬡' relational patterns was thought to be the constants in y=2x+1 and y=2x+2 expressions. That is to say, Zeynep noticed the constants and focused them and could solve them. In '◉ ; ◉◉ ' relational pattern appropriate to 'y=2x' expression, she controlled the options and found correct answer. Similarly, in this pattern type she thought that the option she chose could not be longer than the last step, with the help of this thought she could reach correct answer.

In this study four participating students' algebraic thinking processes analysed with respect to three pattern types. However, for being able to generalize a study, only one pattern type can be chosen and with the help of more students. Besides, when a study is thought to be done with preschool students, instead of choosing from options, children could draw their answers and express themselves.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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