Measurement Science and Technology

PAPER • **OPEN ACCESS**

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To cite this article: Meifang Wu *et al* 2023 *Meas. Sci. Technol.* **34** 095005

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IOP Publishing
Measurement Science and Technology
Meas. Sci. Technol. 34 (2023) 095005 (14pp)
https://doi.org/10.1088/1361-6501/acd545

Relativistic effects of LEO satellite and its impact on clock prediction

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Received 22 February 2023, revised 9 May 2023
Accepted for publication 12 May 2023
Published 26 May 2023

Abstract

Low Earth orbit (LEO) augmentation in the global navigation satellite system has become a focus in the current satellite navigation field. To achieve high precision in positioning, navigation and timing services, relativistic effects should be considered, as they are difficult to distinguish from LEO satellite clock estimates and disturb their predictions. The relativistic effects on LEO satellite clocks are discussed in detail based on both theoretical and empirical results. Two LEO satellite clock prediction strategies are proposed, with and without removing the relativistic effect, using real data from typical LEO satellites: SENTINEL-3B and Gravity Recovery and Climate Experiment Follow-On (GRACE FO-1). For GRACE FO-1 and SENTINEL-3B, the relativistic effects are both on the order of nanoseconds and after removing the relativistic effects, the modified Allan deviations of the clocks are shown to be significantly improved. Based on the prediction strategies proposed, for SENTINEL-3B at around 810 km, with the prediction period increased from 30 to 3600 s, the root mean square error (RMSE) increases from 0.025 ns to about 1.4–1.6 ns. For the lower LEO satellite GRACE FO-1 at around 500 km, the RMSE of the predicted clocks increases more rapidly, i.e. from 0.012 ns at 30 s to about 4.5 ns at 3600 s. Results showed that the LEO satellite relativistic effects developed based on the theory could correct the majority, but not all of the once- and twice-per-revolution terms in the LEO satellite clocks. Although the corrections have exhibited effective improvements in the clock stability, they do not behave better than simply applying the mathematical model to the clock predictions. The latter model, however, does not have physical foundations as the former one.

Keywords: LEO, satellite clock, relativistic effect, clock prediction, GNSS

(Some figures may appear in colour only in the online journal)

1. Introduction

Currently, the global navigation satellite system (GNSS) needs the augmentation of other technologies and systems to fulfill the increasing needs of diverse positioning, navigation and time (PNT) services [1]. This includes, e.g. integrating inertial navigation systems, and the augmentation of low Earth orbit (LEO) satellite navigation systems. In recent years, the latter augmentation has become a hot topic in both the research and industrial fields. With the completion of different LEO constellations such as Space X, Boring, OneWeb, 'Hongyan' and 'Hongyun', the research on LEO satellites has entered a period of vigorous development [2].

Due to the much lower orbital heights and the resulting shorter signal transmission distances compared to the GNSS satellites, LEO satellites exhibit significantly improved signal strength and anti-jamming/deception performances. The lower

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orbital height also leads to fast speed and apparent Doppler frequency shift, which is helpful in improving the accuracy of the velocity measurements and the performance of the cycle slip detection. Moreover, the LEO satellite geometry changes rapidly, which is expected to fundamentally solve the problem of the long convergence time in the precise point positioning (PPP), and enable fast ambiguity resolution and fast convergence, especially in harsh environments [3]. LEO satellites are expected to supplement the medium or higher orbit constellations of GNSS and would have significant advantages in enhancing the accuracy, integrity, continuity, and availability of GNSS. As such, the LEO augmentation in GNSS has become a focus in the current satellite navigation field.

Every coin has two sides. At the same time, LEO satellites have some disadvantages due to their low orbits, e.g. the influences of the increased complexity of the air drag effect and the Earth's nonspherical gravity, and also, the more complicated relativistic effect contained in the LEO satellites. These lead to higher challenges in determining and predicting high-precision LEO satellite orbits and clocks. To enable high-performance LEO-augmented PNT services, especially in real-time, it is necessary to guarantee high accuracy in the short-term predicted orbits and clocks. This requires detailed research in the above-mentioned areas. In this contribution, the focus is put on the LEO satellite clocks, more precisely, the influences of the relativistic effects on LEO satellite clock prediction.

The relativistic effect cannot be ignored when determining and predicting satellite clocks in space [2], where the challenges are incredibly high for the satellites flying at LEOs. The relativity causes a frequency shift in the satellite clock, which leads to time-varying corrections. Correspondingly, the periodic frequency shifts result in periodic time corrections which are contained in the estimated LEO satellite clock parameters using GNSS-based determination methods. Different from the GNSS satellites, the relativistic effects of LEO satellites are much more complicated due to their much shorter distances to the Earth. These effects significantly influence the evaluation of the satellite clock stability, and the clock prediction. Furthermore, it also influences satellite-ground time synchronization, which is highly related to the stability of the on-board satellite clocks to make a meaningful comparison between the time kept by a clock on the satellite and that kept by one on the ground.

High-precision crystal oscillator clocks have long been used in space systems for timekeeping purposes. The ultra-stable oscillator (USO), e.g. has good short-term stability reaching the level of 10^{-13} at an averaging time from 1 to 1000 s. They have been used in diverse space missions that require good short-term clock stability, like the Gravity Recovery and Climate Experiment (GRACE) [4], the GRACE Follow-On (GRACE-FO) [5], and the SENTINEL-3 [6]. In addition, atomic and even optical clocks were also proposed to be mounted on the LEO satellites within navigation systems, like the Kepler system initiated by the German Aerospace Center (DLR). Under the disturbances of the relativities effects at a few nanoseconds, the stabilities of these good clocks are strongly disturbed, leading to possible waste of such good clock stabilities.

Based on current technologies, LEO satellite clock parameters can be determined in near-real-time with high precision. As stated by Li *et al* [7], the LEO satellite clock can be estimated as an outcome of the reduced-dynamic precise orbit determination (POD) process combining GNSS measurements and dynamic models, while the orbits can reach an accuracy of centimeters [8]. The major problem related to the contents of the determined clock parameters. Firstly, as mentioned by Yang *et al* [9], the LEO satellite clocks determined with GNSS measurements contain the code hardware biases of the GNSS receivers/antennas, but not those from the hardware transmitting signals down to the Earth. This requires careful hardware calibration or changes in the clock determination methods, e.g. based on ground network stations, which is not the focus of this study. Secondly, as mentioned before, the estimated LEO satellite clocks contain relativistic effects that are difficult to be perfectly corrected. This affects the LEO satellite clock prediction, which is essential for real-time ground-based applications. Zhang [2] stated that due to the special dynamic characteristics of LEO satellites when accurately determining and predicting clocks, special attention should be paid to the difference between LEO satellites and traditional GNSS satellites in terms of fitting duration, update frequency, the number of parameters, etc. Wang [10] analyzed the factors influencing the prediction of the LEO satellite clocks and proposed a new prediction model without attempting to correct the relativistic effects, however, based only on mathematical models.

At this stage, as the relativistic effects have become a bottleneck to the LEO satellite clock prediction, one would question how far one can go to correct the LEO satellite relativistic effects, and if these corrections really help with the clock prediction. Various studies have been performed on the relativistic effects of GNSS satellite clocks. Han [11] and Wang *et al* [12] analyzed the BeiDou satellite clocks, including the influences of the Earth's shape and the tidal potentials of the Sun and Moon. Wang *et al* [13] studied the J2 relativistic effect on the performance of on-board GPS and BeiDou Navigation Satellite System (BDS-3) clocks. Kouba [14, 15] and Formichella [16, 17] focused on the improved relativistic transformations in GPS and Galileo, especially the J2 correction. Sun *et al* [18] and Guo *et al* [19] studied the relativistic effects on two-way precise time synchronization. The relativistic effects on the LEO satellite clocks, however, are less studied. Larson *et al* [20] accessed the relativistic effects of GRACE satellites, and concluded that the corrections had raised the power of the first-order effects, possibly due to other influences like the voltage variations. Wang [10] analyzed the periodic items of the GRACE-FO and SENTINEL-3B satellites with mathematical models.

In this contribution, detailed formal and analytical studies are performed about the relativistic effects of the LEO satellite clocks. Digging into the different physical reasons and their influences on the relativistic effects and clock estimates, the test results are shown using real data of two LEO satellites at different altitudes, i.e. the GRACE FO-1 at around 500 km, and the SENTINEL-3B at around 810 km. Afterward, an algorithm is proposed for the LEO satellite clock prediction when correcting the relativistic effects based on theoretical derivations. A comparison is then given for the cases not correcting and correcting the relativistic effects.

The paper begins with a short introduction to clock determination. Next, the formal derivation, estimation, and analysis of relativistic effects for LEO satellites are given in detail, using real data of LEO satellites SENTINEL-3B and GRACE FO-1. Subsequently, a clock prediction strategy is explained, and the results of the clock prediction with and without removing the relativistic effects are compared. The conclusions and discussions are given at the end.

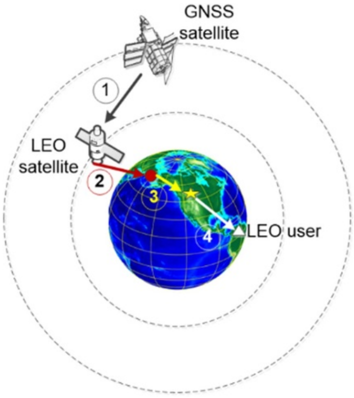


Figure 1. The procedure of LEO satellite clock determination. Note that the location of the monitoring stations and the processing center, and the LEO user are only given as an example for purpose of illustration.

2. LEO satellite clock determination

For applications requiring high-precision clock determination, the reduced-dynamic POD process is often utilized to produce the orbital dynamic parameters and satellite clocks as the output of one round of the least-squares adjustment [21]. Benefiting from the fact that LEO satellites can receive GNSS measurements as a user flying in their orbits, but also under the constraints of the dynamic models, the orbits of LEO satellites can be improved compared to the kinematic mode, namely the PPP mode [22], when combining both information. This is especially beneficial for the highly correlated radial orbits and clocks.

In this study, the ionosphere-free (IF) combination of the GPS code and phase observation on L1 and L2 is used to determine the LEO orbits and the satellite clocks within a least-squares adjustment [23, 24]. The procedure of LEO satellite clock determination is shown in figure 1. The GNSS observations are supposed to be collected the LEO satellites and then transmitted to the ground, more specifically, to monitoring stations (the red dot) and the processing center (the yellow star). After that, zero-difference observation files are processed, introducing precise high-rate real-time GNSS clocks and orbits to estimate the position and clocks of LEO satellite. The first-order ionospheric delays are eliminated by forming the IF combination. The users can obtain estimated LEO satellite clocks and orbits via the Internet, and the LEO-tracked GNSS observations via inter-satellite links and the Internet. The actual latency depends highly on the status of diverse links and the Internet. For data gaps due to technical reasons, or for cases without inter-satellite links, it could take up to hours to allow the transmission of the on-board GNSS data to be ground [24]. As such, the LEO satellite clocks used by real-time ground users, are actually predicted clocks with these latencies bridged.

As typical LEO satellites, GRACE FO-1 and SENTINEL-3B in 2018 are used for processing in this study. GRACE FO-1, launched on 22 May 2018, has an orbital height of about 500 km, an eccentricity smaller than 0.0025, an inclination of about 89° [6], and an orbital period of about 1.6 h in August 2018. SENTINEL-3B, launched on 25 April 2018, has an orbital height of about 810 km, an eccentricity of about 0.0001, an inclination of about 98.65° [7], and an orbital period of about 1.7 h in August 2018.

In this study, the raw GNSS observations at Level-1A are assessed to evaluate the LEO satellite clock behaviors. To achieve possibly high precision of the LEO satellite clock estimates and observe their actual behaviors, GPS final products provided by CODE, such as clocks with a 5 s sampling interval, orbits, and ERP, are used to determine the clocks and orbit of GRACE FO-1 and SENTINEL-3B, respectively. The time reference of the determined LEO satellite clocks is consistent with that of the clock products provided by CODE, which is normally a selected station clock equipped with a hydrogen-maser with high stability.

Since the USO used on the GRACE satellite is free-running, soon after launch, their observations are no longer synchronized with the ground GPS network. The clocks estimated using Level-1A data thus look like a straight line with a relatively large slope, as shown in figure 2(a). The second-order term is also included but is not as clear as the first-order term. After detrending with a quadratic polynomial, the periodic terms of about 1.6 h can be observed above a larger pattern of other systematic effects, as shown in figure 2(b). The larger systematic effects are possibly caused by other external influences like temperature variation [20], which is not the focus of this study. Because the fourth-order polynomial was removed from the timestamps in the observations of SENTINEL-3B, as shown in figure 3, the 1.7 h periodic term is more evident without the need for detrending.

As shown in figures 2(b) and 3, the clocks of GRACE FO-1 and SENTINEL-3B both contain several periodic terms, including mid to long-term periodic items (out of external influences as explained above) and short-term periodic items like relativistic effects. Both of these effects strongly disturb the clock stability compared to the expected behavior of a USO

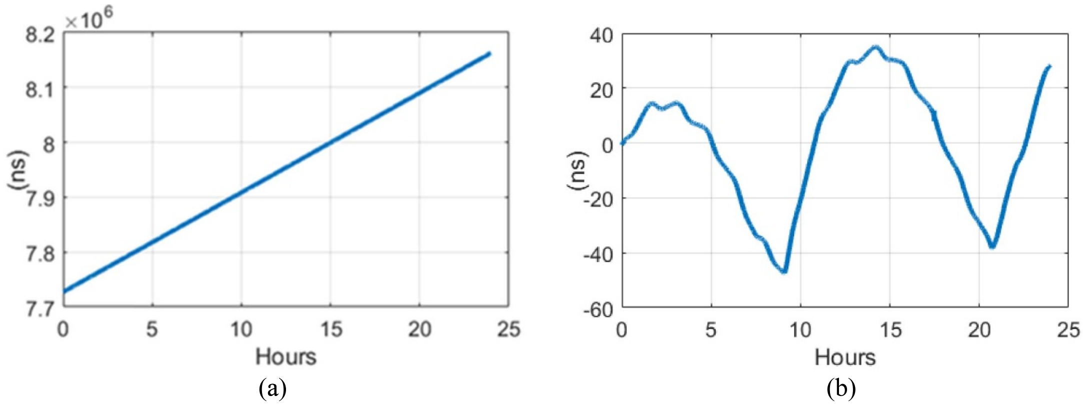


Figure 2. The estimated clocks of GRACE FO-1 on 14 August 2018, using jet propulsion laboratory (JPL) Level-1A data with (a) no polynomial removed and (b) a quadratic polynomial removed.

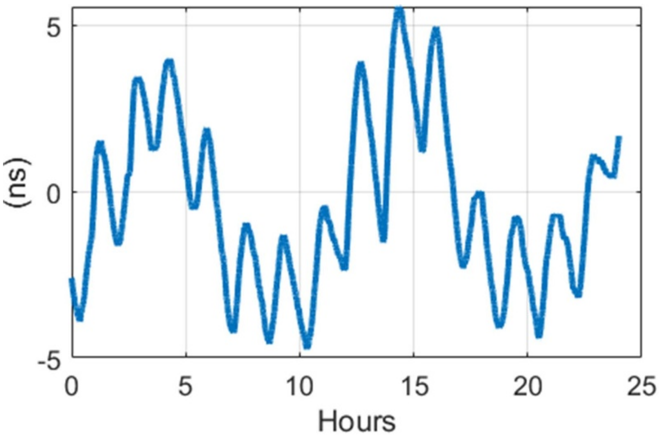


Figure 3. The estimated clocks of SENTINEL-3B on 14 August 2018.

on the ground. Therefore, in order to better reveal the characteristics of the clock itself on the satellite, these effects need to be corrected. In the section 3, the focus is put on the correction of the relativistic effects based on physical models.

3. The relativistic effect of the LEO satellite clocks

In this section, the formula of the relativistic effect for LEO satellite clocks is derived from two different categories: the time dilation and the gravitational redshift. The relativistic effect is then deeply analyzed, including the influence of these two parts separately, as well as the influence of the total relativistic effect on the LEO satellite clocks in the time and frequency domain.

3.1. Estimation of relativistic effects on LEO satellite clocks

The relativistic correction method commonly used in GNSS, which is recommended in IS-GPS-200N [25], can be formulated as follows:

$$\delta\tau = -\frac{2\vec{R}\bullet\vec{V}}{c^2} \tag{1}$$

where $\vec{R}$ is the instantaneous position vector of the satellite, $\vec{V}$ is the instantaneous velocity vector of the satellite. It is noted that equation (1) involve keeping only the leading monopole part of Earth's gravitational potential [14, 17]. So, it is less accurate if applied to LEO satellites. Therefore, a more accurate expression is given in the following part.

As mentioned before, the LEO satellites GRACE-FO and SENTINEL-3B are equipped with USOs, which have good short-term clock stability at the level of 10^{-13}. Good clock stability greatly benefits the PNT service, but it also poses new challenges when dealing with the satellite clocks in orbit. When using the GNSS measurements to determine the LEO satellite clock as described before, the relativistic effects of the LEO satellite clock are not corrected. They are fully contained in the estimated LEO satellite clocks. The relativistic effects of the LEO satellites, which are greatly different from those of the medium Earth orbit satellites or Geostationary satellites, should be processed carefully.

As well known, the relativistic effects in satellite clocks can be split into two different categories: time dilation and gravitational redshift. The time dilation can be formulated as equation (2). Compared with a clock at rest in the Earth Center Inertial Coordinates (ECI) frame, the fractional frequency shift of a clock moving with velocity is [13]:

$$\frac{\Delta f}{f} = -\frac{1}{2}\frac{v^2}{c^2} \tag{2}$$

where v is the magnitude of a clock moving velocity, and c is the vacuum speed of light, f is the frequency of a clock, Δf is the frequency shift.

The second relativistic effect is the gravitational redshift [11, 13, 14, 17], expressed in terms of W, which is the potential difference between the satellite clocks in orbit and clocks on the surface of the Earth

$$\frac{\Delta f}{f} = \frac{W}{c^2}. \tag{3}$$

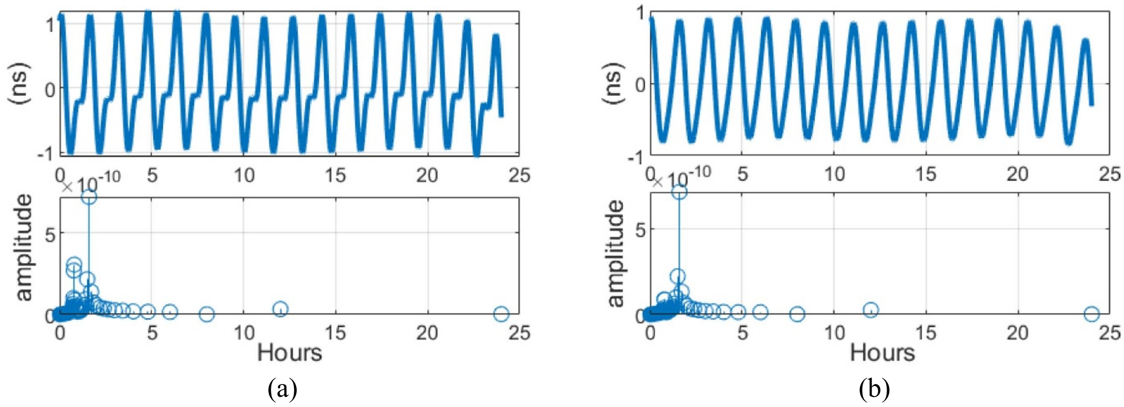


Figure 4. The time dilation (a) and the gravitational redshift effect (b) of the GRACE FO-1 clock and their fast Fourier transform (FFT) results on 14 August 2018.

Adding equations (2) and (3), the whole frequency shift caused by the relativistic effect is:

$$\frac{\Delta f}{f} = -\frac{1}{2}\frac{v^2}{c^2} + \frac{W}{c^2}. \tag{4}$$

Let τ be the proper time and t be the coordinate time, one has then the following expression:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = 1 + \left(\frac{W}{c^2} - \frac{1}{2}\frac{v^2}{c^2}\right). \tag{5}$$

The potential difference, including the gravitational potential of the Earth and the tidal potential caused by the other celestial bodies, such as the Sun and Moon, represented by W_E, W_S and W_M respectively, can be expressed with equations (6)–(9)

$$W = W_\mathrm{E} + W_\mathrm{S} + W_\mathrm{M} \tag{6}$$

$$W_\mathrm{E} = -\frac{\mu_\mathrm{E}}{r}\left[1 - J_2\left(\frac{a_\mathrm{E}}{r}\right)^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\right] \tag{7}$$

$$W_\mathrm{S} = -\frac{\mu_\mathrm{S}}{r_\mathrm{SE}^3}r^2\left(\frac{3}{2}\cos^2\theta_\mathrm{S} - \frac{1}{2}\right) \tag{8}$$

$$W_\mathrm{M} = -\frac{\mu_\mathrm{M}}{r_\mathrm{ME}^3}r^2\left(\frac{3}{2}\cos^2\theta_\mathrm{M} - \frac{1}{2}\right) \tag{9}$$

where μ_E represents the gravitational constants of the Earth, and μ_S, μ_M represent the gravitational constants of the Sun and the Moon, respectively. θ is the polar angle (or colatitude) of the satellite, θ_S and θ_M are the intersection angle between the satellite and the Sun and the satellite and the Moon, respectively. r is the the geocentric distance, r_SE and r_ME represent the distance between the Sun and the Earth and the distance between the Earth and the Moon. a_E is the semi-major axis of the Earth's equatorial plane. J_2 is the oblate Earth's gravity coefficient.

Combing equations (5)–(9), where μ_S is 1.327×10^{20}, μ_M is 4.903×10^{12}, r_SE is about 1.5×10^{11}, r_ME is about 3.8×10^8, it can be inferred that for LEO satellites, the relativistic effects caused by the Sun and Moon on the frequency of satellite clock is at the level of $10^{-18} - 10^{-19}$. Compared with the highest precision of on-board satellite clocks (10^{-15}), it is so tiny that it can be safely ignored. Combing equations (5) and (7), and performing the integration over coordinate time t, the relativistic effect for the LEO satellite clock, named $\delta\tau$, can be described as:

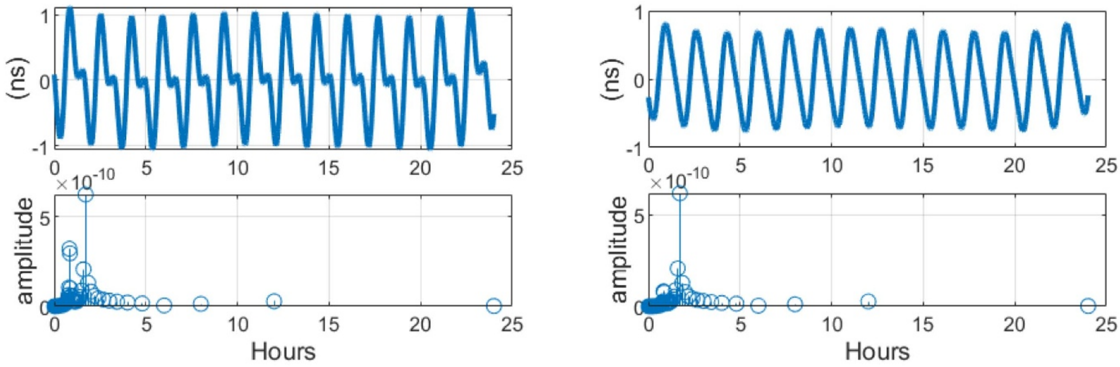
$$\delta\tau = -\int\frac{\mu_\mathrm{E}}{c^2 r}\mathrm{d}t + \frac{\mu_\mathrm{E}J_2}{c^2}\int\frac{a_\mathrm{E}}{r^3}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\mathrm{d}t - \frac{1}{2c^2}\int v^2\mathrm{d}t. \tag{10}$$

3.2. Analysis of relativistic effects on LEO satellite clocks

Using the LEO satellite clock determined in section 2, the relativistic effects of LEO satellite clocks are estimated according to the formulas in section 3.1 and analyzed. Data from GRACE FO-1 and SENTINEL-3B on 14 August 2018, are used as before.

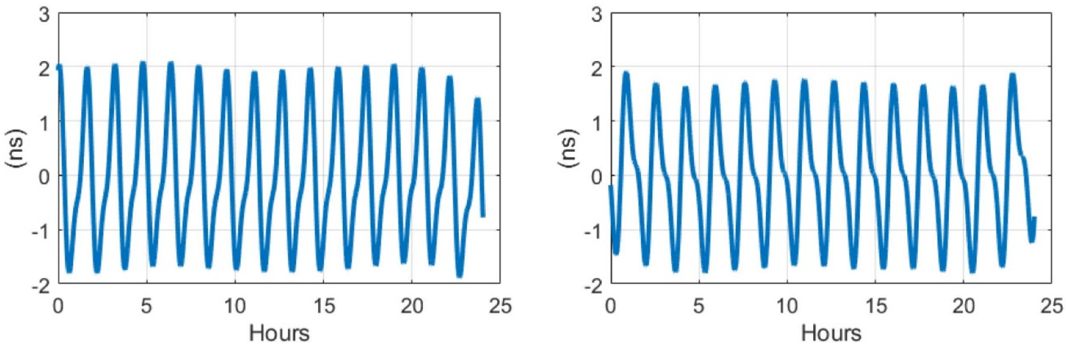
In equation (10), according to the previous deduction, the sum of the first and second terms are called in the following contexts the relativistic effect caused by gravitational redshift, and the third term is the time dilation. Figure 4 shows these two effects on GRACE FO-1 satellite clock and their FFT result, respectively. Figure 5 shows these two effects on the SENTINEL-3B satellite clock. In order to emphasize the periods in the figures 4 and 5, a fifth order polynomial has been estimated and removed firstly from the gravitational redshifts and the time dilations.

From these two figures it can be seen that both the time dilation and the gravitation redshift effect on the LEO satellite clocks are strongly related to the orbital period. This is consistent with their formulas, as they are the integral of v and r with respect to t, respectively. The time dilations on both satellite clocks have parts greater than 1 ns, and the influence of gravitational redshift is slightly smaller, but is also on the order of ns, which cannot be ignored. Besides, the influence of the time dilation on the satellite clocks seems wobbly, as it is the superposition of once-per-revolution (1/rev) and twice-per-revolution (2/rev) periodic terms. Besides, the effect caused by



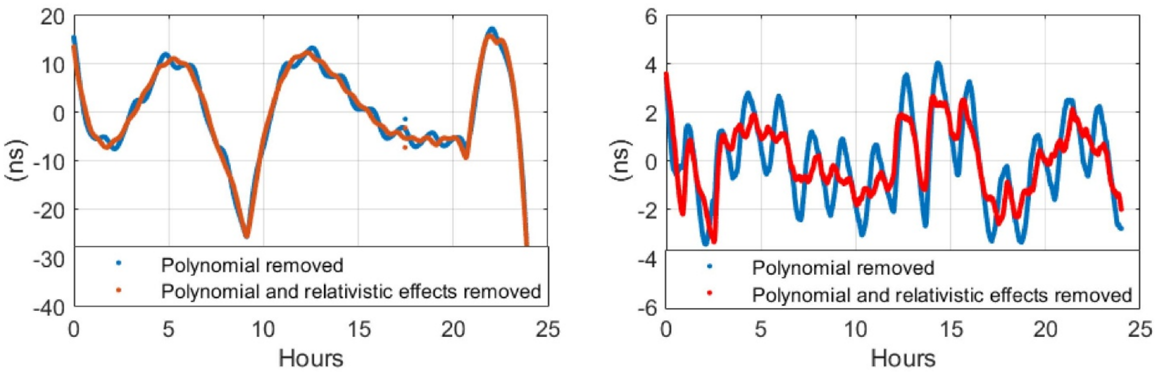
(a) (b)

Figure 5. The time dilation (a) and the gravitational redshift effect (b) of the SENTINEL-3B clock and their FFT results on 14 August 2018.



(a) (b)

Figure 6. The total relativistic effect on LEO satellite clocks on 14 August 2018, for (a) GRACE FO-1, and (b) SENTINEL-3B.



(a) (b)

Figure 7. The LEO satellite clocks on 14 August 2018 before and after removing the relativistic effect for (a) GRACE FO-1, and (b) SENTINEL-3B.

gravitation redshift is also related to the 1/rev and 2/rev periodic terms, as seen in figure 4(b), but the amplitude of 2/rev is smaller than that of time dilation.

Figure 6 shows the total relativistic effect of GRACE FO-1 and SENTINEL-3B, namely the sum of the time dilation and the gravitation redshift effect on the clocks (a fifth order polynomial has been estimated and removed). It can be observed that the relativistic effect has a slightly larger influence on GRACE FO-1, compared to that on SENTINEL-3B. This is related to the different orbital heights of the satellites.

In addition, the changes in the LEO satellite clock before and after removing the relativistic effect are also compared. In order to see the changes in the LEO satellite clocks more clearly, a fifth-order polynomial is removed before the comparison, as shown in figure 7. It can be observed that the correction of the relativistic effect makes the LEO clocks

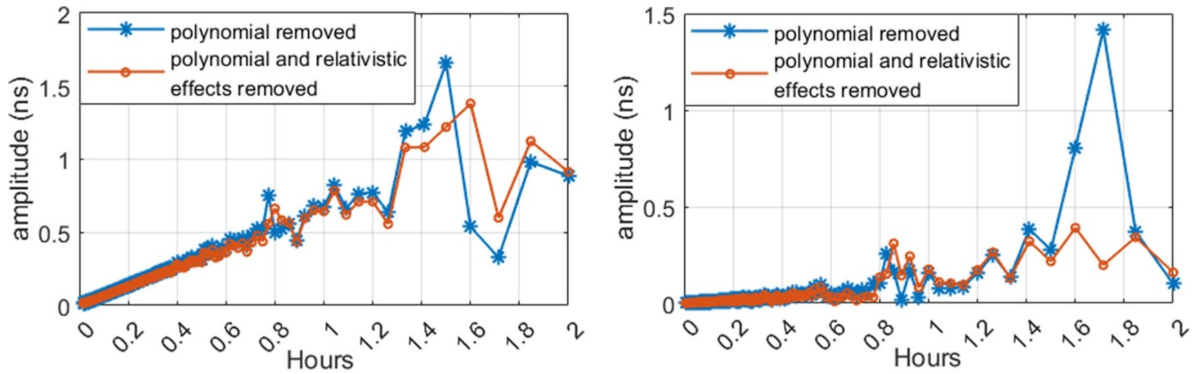


Figure 8. The FFT result of the LEO satellite clocks on 14 August 2018 before and after removing the relativistic effect for (a) GRACE FO-1, and (b) SENTINEL-3B.

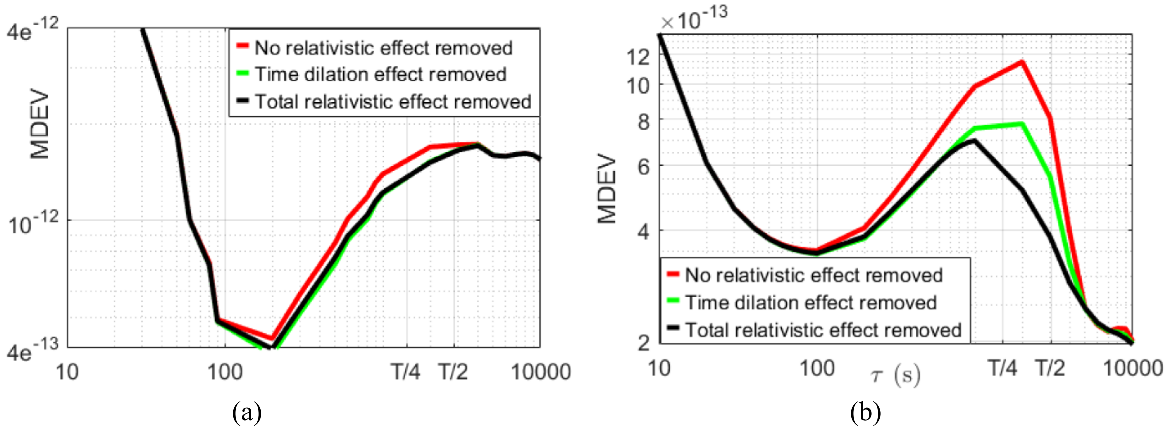


Figure 9. MDEVs of LEO satellite clocks on 14 August 2018 before and after removing the relativistic effect for (a) GRACE FO-1, and (b) SENTINEL-3B.

smoother. The fluctuation range of the clock also becomes smaller. Figure 8 shows the FFT result of the LEO satellite clocks before and after removing the relativistic effect. To focus on 1/rev and 2/rev periodic term related to relativistic effects, the *x*-axis of figure 8 is 0–2 h. It can be seen that, for SENTINEL, the amplitudes of the 1/rev periodic terms has decreased when relativistic effects are removed, and the amplitudes of the 2/rev periodic terms have slightly increased. For GRACE FO-1, there's a slight increase in the amplitudes of the 1/rev and 2/rev periodic terms. The increase in amplitudes may be due to the larger systematic effects possibly caused by other external influences.

The modified Allan deviation (MDEV) of the LEO satellite clocks before and after removing the relativistic effect are also compared. As shown in figure 9, for both satellites, removing relativistic effect improves the MDEVs of the satellite clock, especially when the averaging time is greater than 100 s. This verifies that correcting the relativistic effects effectively improves the mid-term stability of the LEO satellite clocks.

For GRACE FO-1, with only the time dilation removed (see the green line in figure 9(a)) and all the relativistic effects removed (see the black line), the improvements in MDEV are similar for τ between 100 s and 1000 s. This suggests that the redshift effects do not significantly influence the short- to mid-term stability of the clocks. The time dilation plays a major role when comparing the red and the green lines. However, with the increase of τ to greater than 1000 s for SENTINEL-3B (see figure 9(b)), the complete removal of the relativistic effect obviously makes the MDEV better. That is because the bump in the Allan deviation due to the J2 periodic component (the second term of equation (10)), is improved while the relativistic effect is removed. As we known, the bump due to the J2 periodic component is at an averaging time of about $T/4$ (T is the orbital period), the bump due to orbital estimation errors is at the averaging time of about $T/2$, and the two bumps may appear as a single deformed bump, located at between about $T/4$ and $T/2$ [16, 26], for GARCE FO-1 and SENTINEL-3B, is about 1000–3000 s. From figure 9(b), it can be seen more clearly that MDEV is improved significantly when τ is between 1000 s and 3000 s. The very similar black and green lines for GRACE FO-1 (figure 9(a)) could be caused by very large systematic effects caused by external reasons, making the relativistic effects less significant. The correction of both relativistic effect terms is important to recover the clock stability, especially in mid- to long-term.

The relativistic effects result in 1/rev and 2/rev periodic variations in the space-borne clocks. In figure 10, 1/rev and 2/rev periodic variations are calculated based on mathematical

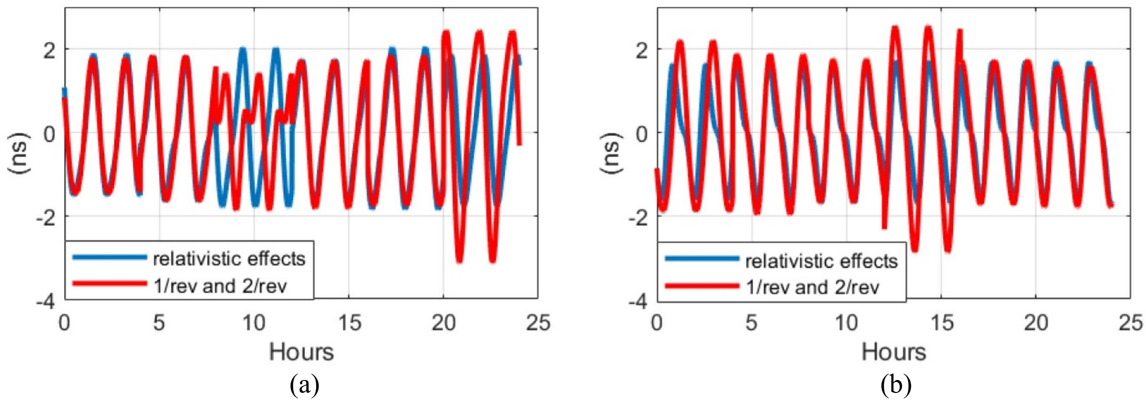


Figure 10. Comparison of the mathematical 1/rev and 2/rev periodic variations and relativistic effects of the LEO satellite clocks on 14 August 2018, for (a) GRACE FO-1 and (b) SENTINEL-3B.

fitting (red) and calculated with relativistic effects (blue) of the GRACE FO-1 and SENTINEL-3B clocks according to equation (10) are compared. Mathematical fitting refers to analyzing the LEO satellite clocks by finding the parameters related to 1/rev and 2/rev periodic terms using the FFT. It can be seen from figure 10 that the cycle and the trend of the two curves match well with each other. The relativistic effect calculated according to equation (10) has corrected the majority of the 1/rev and 2/rev periodic variations. On the one hand, this verifies the correctness of using equation (10) to estimate the relativistic effect of LEO satellite clocks. On the other hand, there're remaining terms in the 1/rev and/or 2/rev patterns, which are possibly caused by other effects unknown to us.

4. The effect of relativistic effect on LEO satellite clock prediction

LEO satellite is expected to provide users with high-precision real-time services in the future, and among different LEO real-time products, LEO satellite clocks are essential. However, no matter which method is used to determine the LEO satellite clocks, the latency of the clock products exists. Therefore, the gap between the usage time and the product calculation time has to be bridged by predicting the satellite clocks. Thanks to the good stabilities of these frequency oscillators LEO satellites, it has offered the possibility of short-term clock prediction with acceptable precision. Besides, the relativistic effect with periodic changes can also be predicted with different methods. In this section, two kinds of LEO clock prediction algorithms are designed and discussed.

A lot of studies have been performed for GNSS clock prediction. Polynomial models were used for the International GNSS service-real time service (IGS RTS) clock prediction in the short term of a few minutes and for the broadcast clocks in the navigation message for a longer term of up to a few hours [27, 28]. A quadratic polynomial plus two periodic fitting and Back Propagation (BP) neural network model was proposed to predict the BDS ultra-rapid satellite clocks of an hour to a day [29]. In order to overcome the problems of data stream interruption and delay in broadcasting clocks in real-time applications, a model consisting of linear polynomial and periodic terms was used to predict the GPS clocks for minutes to hours[30–32]. In contrast, the prediction of LEO satellite clocks is less studied. According to Wang [10], LEO satellite clocks could be described as the sum of the polynomial and periodic terms due to the systematic effect on the LEO satellite clocks. In this section, the model employing the polynomial fitting and the periodic terms is used to predict the LEO satellite clocks, distinguished between the cases with and without removing the theory-based relativistic effects.

Using the FFT to analyze the time series of the LEO satellite clocks on August 14–20, 2018. It can be observed that for both the GRACE FO-1 or the SENTINEL-3B, there are several periodic terms overlapped in the clocks after removing the polynomial. The dominant mid- to long-term periodic effects in GRACE FO-1 have a period of 12 h and 6 h. For SENTINEL-3B, the dominated mid- to long periodic effects have a period of about 12 h. Figure 11 shows the mid- to long-term periodic effect of LEO satellites on 14 August 2018.

As seen from figure 11, in addition to the mid- to long-term periodic terms, the LEO satellite clocks also contain other short-term periodic terms. As previously mentioned, the relativistic effect is strongly related to the LEO satellite orbital period. They are, for this reason, typically named the 1/rev and 2/rev periodic variations. As such, in addition to the mid- to long-term period terms determined by the FFT, the orbital period and half of the orbital period are considered for modeling the short periodic terms in LEO satellite clocks.

To sum up, the LEO satellite clocks can be described as a sum of periodic terms and polynomials, as seen in equation (11)

$$\hat{C}lk(t_i - t_0) = \hat{a}_0 + \hat{a}_1(t_i - t_0) + \cdots + \hat{a}_m(t_i - t_0)^m + \sum_{j=1}^{k} \hat{A}_j \sin\left(\frac{2\pi}{\hat{T}_j}(t_i - t_0) + \hat{\varphi}_j\right) \quad (11)$$

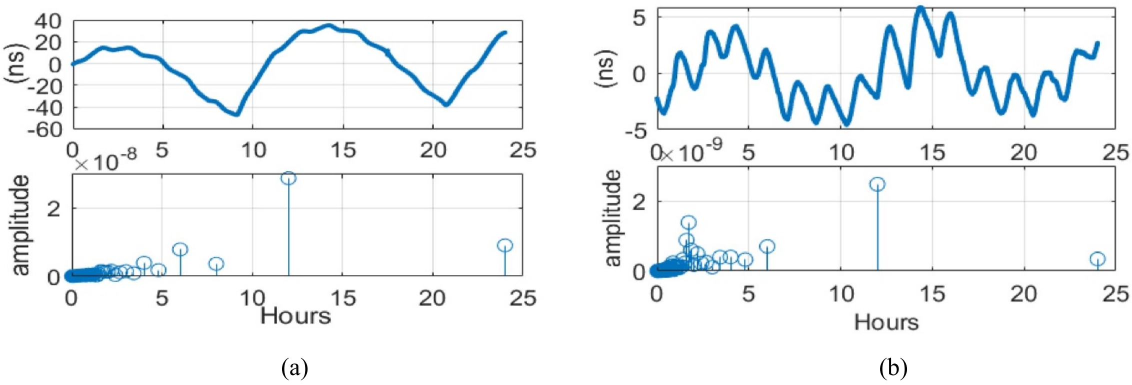


Figure 11. The LEO satellite clocks and their FFT on 14 August 2018, for (a) GRACE FO-1, and (b) SENTINEL-3B.

where $\hat{Clk}$ denotes predicted clocks, t_i denotes predicted epoch, t_0 is the initial epoch, $\hat{a}_i\,(i = 1, \ldots, m)$ is the polynomial fitting coefficient, k denotes the number of the periodic terms, $\hat{T}_j$ is the periods of the periodic terms. $\hat{A}_j$ and $\hat{\varphi}_j$ denote the amplitude and phase of the periodic terms, respectively.

According to equation (11), two different prediction algorithms are proposed. Algorithm 1 considers the mid-to-long periodic terms by analyzing the LEO satellite clocks, and the short-term periodic terms by analyzing the relativistic effect estimated according to equation (10). Algorithm 2 considers all the systematic effects by analyzing the LEO satellite clocks, including the long-to-mid terms and the short terms related to the orbital period, without the correction of the relativistic effect at all. Both algorithms are described in detail, with the test results discussed afterward.

Three scenarios of algorithm 1 are designed, shown in table 1. The analysis started from scenario A of algorithm 1. The process of scenario A of algorithm 1 can be described as shown in figure 12 and steps 1–6.

- Step 1: Load the LEO satellite orbits, and estimate the relativistic effects according to equation (10);
- Step 2: Load the LEO satellite clock estimated in section 2, and use the FFT to analyze the periods of the mid- to long-term periodic terms $\hat{T}_j\,(j = 1, 2)$ with a long fitting interval of 24 h. In this manuscript, on 14 August 2018, for GRACE FO-1, $\hat{T}_j\,(j = 1, 2)$ is 12 h and 6 h, respectively; for SENTINEL-3B, $\hat{T}_j\,(j = 1)$ is 12 h, which is consistent with the figure 11.
- Step 3: Use the estimated periods of the mid- to long-term periodic terms $\hat{T}_j\,(j = 1, 2)$, estimate the amplitude and phase of the mid- to long-term periodic terms $\hat{A}_j\,(j = 1, 2)$ and $\hat{\varphi}_j\,(j = 1, 2)$ with a quadratic polynomial using the same fitting interval as in Step 1. In this manuscript, on 14 August 2018, for GRACE FO-1, $\hat{A}_j\,(j = 1, 2)$ is about 30 ns and 6 ns, respectively; for SENTINEL-3B, $\hat{A}_j\,(j = 1)$ is 2.2 ns.
- Step 4: Set the $\hat{T}_j(j = 3, 4)$ as the LEO satellite orbital period, and estimate the amplitude $\hat{A}_j\,(j = 3, 4)$ and phase $\hat{\varphi}_j\,(j = 3, 4)$ by analyzing the relativistic effects estimated in Step 1 with a fitting interval of 4 h. In this manuscript, on 14 August 2018, for GRACE FO-1, $\hat{T}_j(j = 3, 4)$ is 1.6 h and 0.8 h, $\hat{A}_j\,(j = 3, 4)$ is about 1.7 ns and 0.4 ns, respectively; for SENTINEL-3B, $\hat{T}_j\,(j = 3, 4)$ is 1.7 h and 0.85 h, $\hat{A}_j\,(j = 3, 4)$ is about 1.5 ns and 0.6 ns, respectively.
- Step 5: Estimate $\hat{a}_m$ with $\hat{T}_j$, $\hat{A}_j$, $\hat{\varphi}_j$, m can be set according to the satellite used, maybe 1 or 2. All coefficients are now estimated and the model is determined.
- Step 6: Different fitting intervals and sliding windows are used to test for each prediction interval. For a certain prediction interval, the fitting interval delivering the smallest RMS of the prediction errors is considered the best suitable one. Here the prediction errors stand for the difference between the predicted clocks and the clocks post-estimated according to section 2.

Table 1. Scenarios of algorithm 1.

Scenario	Differences from scenario A
A	—
B	Step 2: Load the LEO satellite clock estimated in section 2, remove the relativistic effects estimated in step 1 first, then use the FFT to analyze the periods of the mid- to long-term periodic terms with a long fitting interval of 24 h;
C	Step 4: Replace the fitting interval of 4 h with 24 h;

Using GRACE FO-1 and SENTINEL-3B clocks on August 14–15, 2018, estimated as described in section 2, the three scenarios of algorithm 1 are tested and analyzed. Only short-term (the prediction period is less than or equal to an hour) is studied here in view of the relevant application scenarios of the LEO satellite clocks. The error budget of the LEO clock prediction errors is summarized in table 2.

It can be seen from table 2 that scenario A has the smallest RMSE of the predicted clocks. In scenario B, the relativistic effects are removed before estimation of the period terms using the FFT, which may lead to a worse analysis of the characteristics of LEO satellite clocks. In scenario C, the too-long fitting interval may result in inaccurate satellite clock fitting. As such, scenario B and scenario C has poorer prediction precision compared with scenario A.

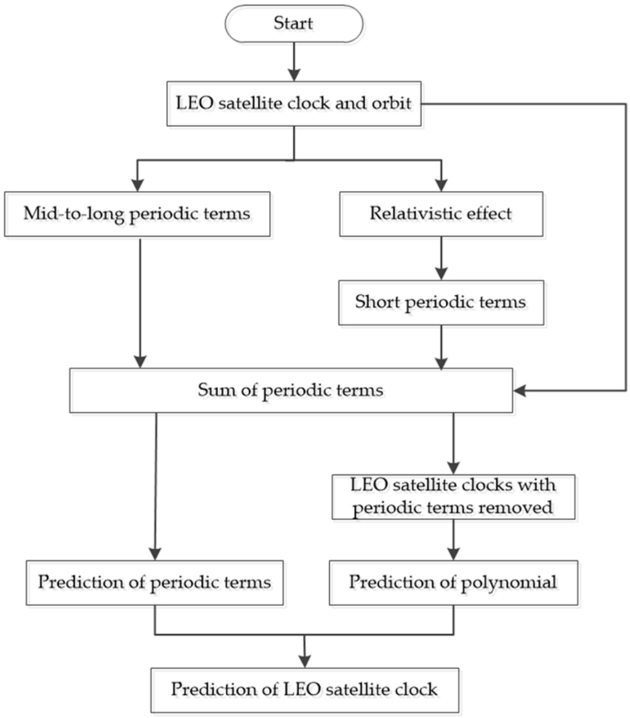


Figure 12. The flowchart describing the process of scenario A of algorithm 1.

Table 2. RMSE of the predicted clocks, and the best suitable polynomial fitting times for different prediction periods based on the three scenarios of algorithm 1.

		RMSE (ns)			Fitting time (s)		
Satellite	Prediction period (s)	Scenario A	Scenario B	Scenario C	Scenario A	Scenario B	Scenario C
SENTINEL-3B	30	0.025	0.025	0.025	50	50	50
	60	0.033	0.033	0.033	80	70	80
	600	0.500	0.500	0.500	130	110	130
	1800	1.233	1.833	1.600	8500	8400	8570
	3600	1.633	1.667	1.667	7950	7540	7610
GRACE FO-1	30	0.012	0.013	0.012	120	60	120
	60	0.018	0.030	0.018	90	60	90
	600	0.300	0.867	0.300	90	180	90
	1800	1.800	4.000	1.933	120	180	120
	3600	4.767	14.333	5.333	180	180	180

As the second algorithm, the process of LEO satellite clock prediction based on the systematic effect can be described as figure 13 and the steps below.

- Step 1: Follow Steps 2–3 of the above process;
- Step 2: Use the fitting residual of the first step here with a fitting interval of 4 h to estimate the amplitude $\hat{A}_j$ $(j = 3,4)$ and phase $\hat{\varphi}_j$ $(j = 3,4)$ of the short-term periodic terms with a fourth-order polynomial. $\hat{T}_j(j = 3,4)$ is set as the LEO satellite 1/rev and 2/rev periodic terms. Here, on 14 August 2018, for GRACE FO-1, $\hat{T}_j(j = 3,4)$ is also set to 1.6 h and 0.8 h, for SENTINEL-3B, $\hat{T}_j$ $(j = 3,4)$ is 1.7 h and 0.85 h. Besides, $\hat{A}_j$ $(j = 3,4)$ is estimated according every fitting interval of 4 h, for GRACE FO-1, $\hat{A}_j$ $(j = 3)$ between 1.2 and 2.5 ns, $\hat{A}_j$ $(j = 4)$ is between 0.15 and 0.8 ns, respectively; for SENTINEL-3B, $\hat{T}_j$ $(j = 3,4)$ is 1.7 h and 0.85 h, $\hat{A}_j$ $(j = 3)$ is between 1.6 and 2.5 ns, $\hat{A}_j$ $(j = 4)$ is between 0.14 and 0.45 ns.
- Step 3: Follow Steps 5–6 of the above process.

It can be inferred from the two processes that the main difference between the two algorithms lies in the determination of the short-term periodic effects. Using GRACE FO-1 and SENTINEL-3B clocks on August 14–15, 2018, estimated as described in section 2, the scenarios A of algorithm 1 (the best one of algorithm 1) and algorithm 2 are compared and analyzed. The error budget of the LEO clock prediction errors is summarized in table 3. Besides, the predicted error of LEO satellite clocks using algorithm 2 is shown in figures 14 and 15, and the 'truth' is the LEO clocks estimated in section 2 using GPS final products provided by CODE.

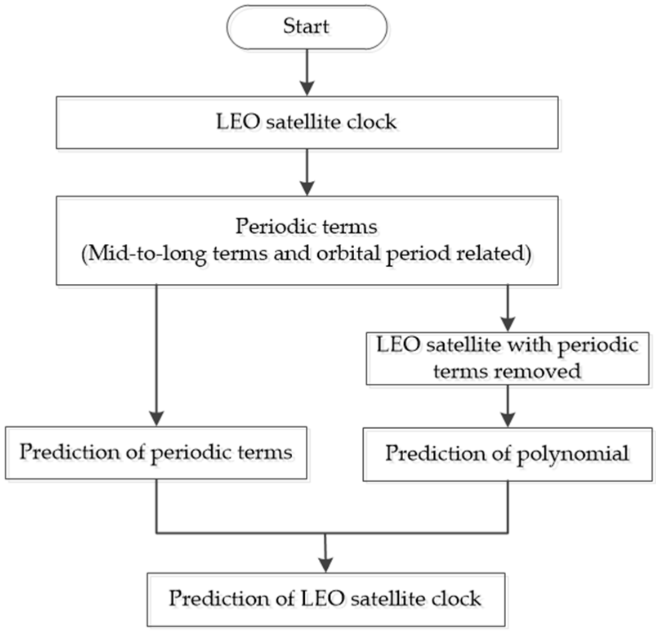


Figure 13. The flowchart describing the process of algorithm 2.

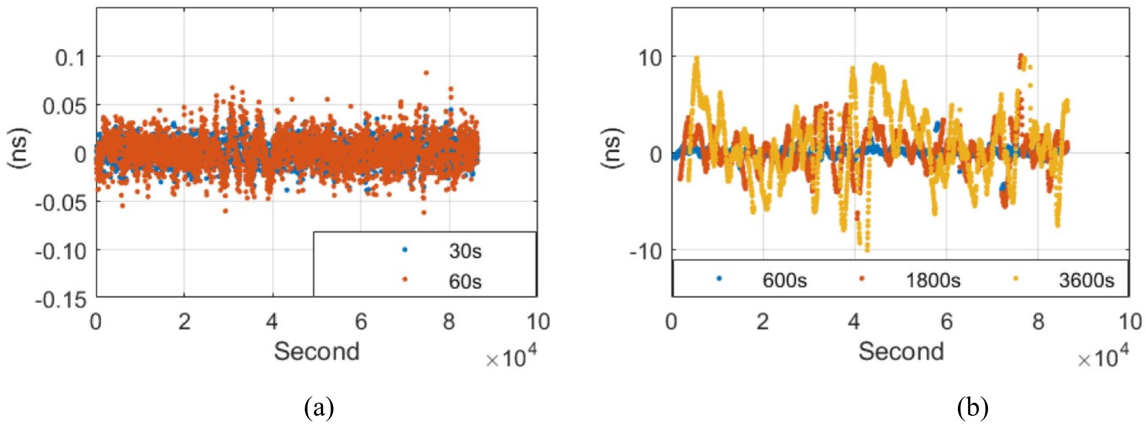


Figure 14. The predicted error of GRACE FO-1 on 14 August 2018, using algorithm 2, (a) for prediction period is 30 s and 60 s, (b) for prediction period is 600 s, 1800 s, and 3600 s.

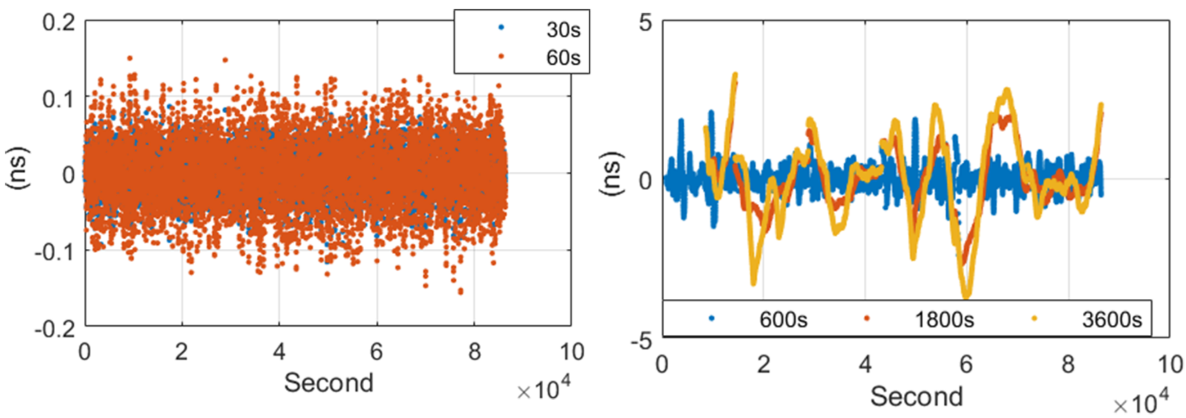


Figure 15. The predicted error of SENTINEL-3B on 14 August 2018, using algorithm 2, (a) for prediction period is 30 s and 60 s, (b) for prediction period is 600 s, 1800 s, and 3600 s.

Table 3. RMSE of the predicted clocks and the best suitable polynomial fitting time for different prediction periods based on the scenarios A of algorithm 1 and algorithm 2.

Satellite	Prediction period (s)	Algorithm 1		Algorithm 2		Benefit
		RMSE (ns)	Fitting time (s)	RMSE (ns)	Fitting time (s)	
SENTINEL-3B	30	0.025	50	0.025	50	0
	60	0.033	80	0.033	80	0
	600	0.500	130	0.433	200	13.3%
	1800	1.233	8500	1.000	8400	18.9%
	3600	1.633	7950	1.362	5000	16.3%
GRACE FO-1	30	0.012	120	0.012	120	0
	60	0.018	90	0.018	150	0
	600	0.300	90	0.333	90	−11.1%
	1800	1.800	120	1.733	120	3.7%
	3600	4.767	180	4.567	180	4.2%

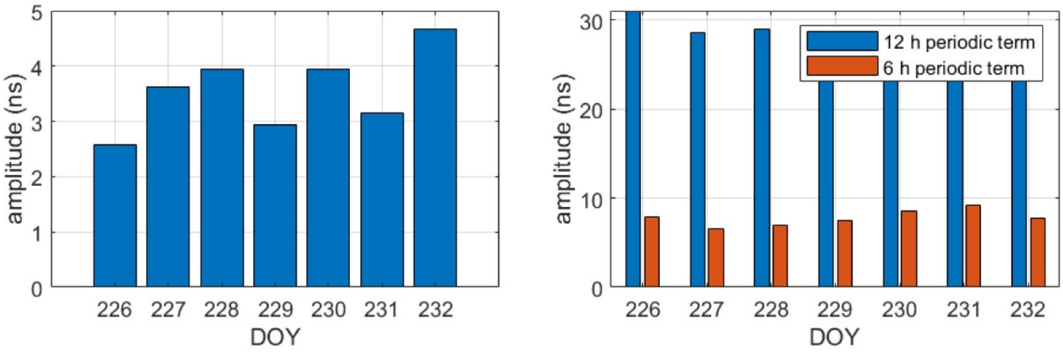


Figure 16. The amplitudes of mid- to long-term periodic effects of GRACE FO-1 and SENTINEL-3B on 14–20 August 2018, for (a) GRACE FO-1, and (b) SENTINEL-3B.

The benefit in the last column of table 3 is calculated with equation (12), i.e. the benefit of predicting the LEO satellite clocks based on the analysis of the systematic effect, compared to that based on the analysis of the relativistic effect:

$$K = \frac{R_\mathrm{r} - R_\mathrm{s}}{R_\mathrm{r}} \tag{12}$$

where R_s denotes the RMSE of the predicted clock errors based on the analysis of systematic effect, and R_r denotes the RMSE of the predicted clock errors based on the analysis of relativistic effect.

As shown in table 3, for SENTINEL-3B, with the prediction period increased from 30 to 3600 s, the RMSE increases from 0.025 ns to about 1.4–1.6 ns. For GRACE FO-1, the RMSE increases more rapidly, from 0.012 ns to about 4.5 ns. There are still huge challenges in predicting clocks for low-altitude satellites like GRACE FO-1.

From table 3 it can also be observed that for both the satellite clocks of SENTINEL-3B or GRACE FO-1, when the prediction period is less than or equal to 60 s, the RMSE of the prediction errors based on the two algorithms, is almost the same. With the increasing prediction period, the RMSE of the prediction errors based on algorithm 2 is better than that based on algorithm 1 in most of cases. This suggests that with incomplete removal of the 1/rev and 2/rev effects using the formulas derived for the relativistic effects (see equation (10)), the clock prediction is not getting better compared to the case of not attempting to remove them at all. For good LEO satellite clock prediction, the current correction methods for relativistic effects, even already specified for LEO satellites, are still not good enough to correct all the 1/rev and 2/rev systematic effects and await further improvements. One might need to explore other reasons causing these effects in addition to the relativistic effects, and correspondingly investigate their correction methods (with the support of physical foundation) to recover the stability of the estimable clocks in LEO satellites.

From tables 2 and 3, it can be inferred that algorithm 2 is more suitable for predicting the LEO satellites. In algorithm 2, the parameters related to the periodic terms need to be estimated using the corresponding training data series at each time of the prediction, as they are not stationary, but are shown to vary with time.

As mentioned before, the FFT was used to analyze the LEO satellite clocks of a week, and the mid- to long-term periodic effects of GRACE FO-1 have a period of 12 h and 6 h, and for SENTINEL-3B, 12 h. With these period parameters fixed, least-squares adjustment was used to determine the amplitudes of the periodic terms. Figure 16 shows the amplitudes of the mid- to long-periodic terms of GRACE FO-1 and SENTINEL-3B, respectively. The amplitude of the 12 h periodic term of GRACE FO-1 is between 27 and 32 ns, and that of the 6 h periodic term is about 8 ns. The amplitude of the 12 h periodic term of SENTINEL-3B is between 2.6 and 4.7 ns. Small day-to-day variations can be observed. Note that a fourth-order polynomial might have been pre-corrected in the SENTINEL-3B raw observation files to correct the timestamps, which has not been considered here.

5. Discussions and conclusions

The LEO satellites are expected to bring diverse improvements in the traditional GNSS-based PNT services, e.g. in aspects of coverage, landing power, initialization time, signal strength, and reliability. It will serve the future real-time precision positioning users represented by automatic driving and unmanned aerial vehicles due to their improved performance. However, LEO satellites have some disadvantages due to their low orbit, such as suffering from a more complicated relativistic effect remaining in the estimable satellite clocks.

The principle and flow of the LEO orbit and clock determination processes were reviewed. For typical LEO satellites like GRACE FO-1 and SENTINEL-3B, the estimated clocks are discussed. The estimated clocks include several periodic items, including mid- to long-term periodic terms and short-term periodic terms.

In order to separate the periodic items from the clock itself, the relativistic effect of the LEO satellite clocks is discussed in detail. The formula of the relativistic effect correction applicable to LEO satellites, including the time dilation and the gravitational redshift, is derived first. Furthermore, the influence of the relativistic effect on LEO satellite clock is revealed. For both GRACE FO-1 and SENTINEL-3B, the time dilation is greater than 1 ns. The gravitational redshift effect is smaller but also at the order of ns. After removing the relativistic effect, the LEO satellite clocks become smoother, and the MDEVs of the clocks are improved. By mathematically analyzing the 1/rev and 2/rev periodic effects in the LEO satellite clocks and comparing them with the relativistic effects estimated, it was found that relativistic effects compensate for the majority of, but not all of the 1/rev and 2/rev periodic variations in the clocks.

Using the FFT to analyze the periodic terms in the LEO satellite clocks, two different prediction methods were proposed and tested. The difference between the two methods lies in whether to determine the short-term periodic terms based on relativistic effect or not. As seen from the prediction results, for SENTINEL-3B, with the prediction period varying from 30 s to 3600 s, the RMSE increases from 0.025 ns to about 1.4–1.6 ns. For GRACE FO-1, the RMSE increased from 0.012 ns to about 4.5 ns. By comparing the prediction results based on the two different methods, it is shown again that the periodic term related to the orbital period contains not only the relativistic effect but also the remaining terms unknown to us so far.

This research attempts to correct the 1/rev and 2/rev based on physical models to the largest extent. The contributions of this work include:

- Investigate how far the relativistic effects within LEO satellites can be modeled based on physical models;
- Compare the algorithm and the clock prediction with and without correcting the relativistic effects based on physical models;
- Compare the variation of parameters in algorithm 2 and give a reference when using algorithm 2 to predict LEO satellite clocks.

As a short summary, the LEO satellite clock estimation, the correction of the LEO satellite relativistic effects, and the short-term prediction of LEO satellite clocks correcting and not correcting the relativistic effects were discussed in this contribution. On this basis, it can be concluded that the current correction formulas developed for LEO satellite relativistic effects can correct the majority, but not all of the 1/rev and 2/rev terms in the LEO satellite clocks. Although the corrections have exhibited effective improvements in the clock stability, they do not improve the clock prediction much due to the remaining terms. Reasons for the uncorrected systematic effects in the LEO satellite clocks are expected to be investigated in future studies.

Data availability statement

The data of GRACE FO-1 were obtained from the JPL. The data of SENTINEL-3B were obtained from the ESA. The GNSS final products were obtained from the Center for Orbit Determination in Europe (CODE).

Acknowledgments

We acknowledge the support of the international GNSS monitoring and assessment system (iGMAS) at the National Time Service Center, and the National Space Science Data Center, National Science & Technology Infrastructure of China (www.nssdc.accn).

Funding

This research was funded by the National Science Foundation of China, Grant Number 12003041, and the National Time Service Center, Chinese Academy of Sciences (CAS) (No. E167SC14), and Key R&D Program of Shaanxi Province (2022GY-099).

Conflicts of Interest

The authors declare no conflict of interest.

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