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On Linear and Non-Linear Approximation In the Theory of Convective Heat Transfer

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Authors' contributions

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

A system of linear equations that is currently widely used to describe convective heat transfer does not seem to be able to explain some experimental facts. One of the reasons for this may lie in using Newton's and Fourier's linear laws when deriving energy and Navier-Stokes equations. Replacing linear equations with nonlinear ones, as well as using an expression for surface heat flux density that is based on laws of physics instead of expressions called 'cooling laws,' would allow to solve a wider range of problems, and also would better agree with the experimental data. The use of proposed non-linear system of equations would also permit engineers in chemical, textile, defense, power, and other industries to design more economical and smaller-sized heat exchange devices.

Keywords: Convective heat transfer; turbulent flows; navier-stokes equation; non-linear equations.

1. INTRODUCTION

At present, theory of convective heat transfer uses a system of linear equations (Navier-Stokes, continuity, energy, and an expression called 'Newton's law of cooling' in English-language literature, or 'Newton-Richmann's law' in Russian-language literature). The results of engineering calculations using these equations are often not consistent with the experimental

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data and principles of thermodynamics, even in the simplest cases of heat transfer. This is why engineers who design and make heat-exchange devices do not trust theoretical calculations, and require experiments instead. If it is impossible for some reason, they overestimate calculated heat transfer surface by 30-50%.

This established practice leads us to believe that currently used system of linear equations may be not completely accurate. The purpose of this paper is to show that it is possible to modify this system to make the results of calculations better and more accurate.

2. LINEAR MODELS OF CONVECTIVE HEAT TRANSFER AND SELECTED SOLUTIONS

Linear models of convective heat transfer proved to be convenient in terms of calculations. However, their validity is usually limited to a small change in important heat transfer parameters, and, for various reasons, they turn out to be not the most accurate. This is also true for convective heat transfer equations.

In the simplest case, when heated fluid is incompressible and mass forces are absent, the system of linear equations for the convective heat transfer can be written in a form of eq. (1-4):

$$\frac{d\bar{v}}{dt} = \frac{\partial \bar{v}}{\partial t} + \bar{v}\nabla v = -\frac{1}{\rho}\nabla P + \frac{\eta}{\rho}\Delta\bar{v} \qquad (1)$$

$$div\bar{v} = 0 \qquad (2)$$

$$q = \alpha\,(T_w - T) \qquad (3)$$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \bar{v}\nabla T = \frac{\lambda}{\rho C}\Delta T \qquad (4)$$

Here $\bar{v}$ – velocity vector; t – time; ρ – fluid density; P – pressure; η – coefficient of dynamic viscosity; ∇ and Δ – Hamilton and Laplace operators; λ – fluid's thermal conduction coefficient; C – fluid's specific heat capacity at constant volume; q – heat flux density (heat transfer); α – heat transfer coefficient (heat loss coefficient); T – fluid temperature (in International System of Units (SI)).

At present time, the system of equations (1-4) is widely used for solving various heat transfer problems, regardless of fluid flow mode or heat transfer channel size. Some researchers [1] believe that because these solutions agree so well with the experimental data, it doesn't make sense to doubt general applicability of the system equations. However, there are some experimental facts that cannot be explained within the framework of this theory. For example, when fluid flow mode in channels changes from laminar to turbulent, its velocity profile becomes more filled, i.e. tends to be more homogeneous. As a result, fluid flow through the channel cross-section increases, and the same happens with temperature profile at the channel cross-section. But existence of different velocity and temperature profiles does not follow from Navier-Stokes linear and energy equations [2].

Also, in the system of equations (1-4) fluid temperature T is interpreted in different ways. In the energy equation (4) temperature is defined as a local value that changes in space and time. But in the expression (3) temperature may mean all kind of different things. Heat transfer textbooks [3-5] define T as a "temperature of a liquid or gaseous medium surrounding an object". Scientific literature elaborates that it can be fluid temperature far from the channel wall, constant fluid temperature at the channel cross-section [6], or an average fluid temperature at the channel cross-section [7]. Some even say that each problem should have its own definition of T: it can be average temperature at channel cross-section, average fluid mass temperature, or constant fluid temperature at the channel cross-section at the inlet of a heated area of the channel. The choice would depend on the nature of the problem, and it would be made only for calculations' convenience [7].

Expression (3) in English-language scientific and technical literature is called 'Newton's law of cooling', and in Russian-language literature it is called 'Newton-Richmann's law of cooling'. But neither Newton, nor Richmann did not use this expression, because at that time they did not yet distinguish between temperature and heat quantity. It was Joseph Fourier who first suggested a concept of surface heat flux density in convective heat transfer and its connection with temperature difference [8]. But his expression (as well as the expression (3)) lacks an important parameter – medium velocity. So, it looks like the expression (3) not only should not be called Newton's (or Newton-Richmann's) law of cooling, but also is not consistent with the first law of thermodynamics [9].

It also seems that theoretical researchers believe that the expression (3) is an "experimental fact," and experimental researchers use it as a theoretical fact. However, experiments do not substantiate linear dependence (3), and it doesn't follow from some other, more general laws of physics [9].

Due to the complexity of solving differential equations in partial derivatives, a linear system can be solved for a small number of specific problems under some simplifying assumptions, dismissing the fact that, while solving Navier-Stokes and energy equations, researchers are trying to determine heat transfer coefficient, which is not a physical value.

Analytical solutions that aim to determine velocity and temperature fields (and also heat transfer coefficient) are called 'precise'. For example, there are known precise solutions to the problem of heat transfer in laminar fluid flow in pipes. This problem was first solved by Leo Graetz in 1883 and 1885. Not knowing about his works, Wilhelm Nusselt solved this problem once again in 1910 [7,6, and 10]. Graetz and Nusselt made the following assumptions: in an absolutely smooth cylindrical pipe with a radius r_0 flows an incompressible fluid in a laminar mode. Average fluid temperature at the inlet of the heated pipe T_0 and wall temperature of the pipe T_w are kept constant throughout the entire fluid heating area, and fluid velocity along the channel cross-section is subject to the Poiseuille distribution law (parabolic velocity profile). The aim of this study was to determine fluid temperature distribution $T(r, x)$ along the radius of r and the length of the pipe x , and to determine how heat transfer coefficient $\alpha(x)$ changes along the pipe length.

Graetz and Nusselt obtained a solution for the modified equation (4) by using separation of variables. As usual in such cases, it was a product of two functions, each of them depending on only one variable. Some difficulties in determining temperature were related to the necessity of superimposing certain conditions when calculating functions in the form of the infinite series.

Graetz and Nusselt concluded that in the fluid layers at small values of the dimensionless quantity of the pipe (near the beginning of the pipe's heated section) fluid temperature at the pipe axis changes only slightly along the pipe radius and pipe length. Only near the pipe wall large temperature changes took place, both along pipe radius r and pipe length x. The area of low values of the dimensionless quantity of the pipe is believed to be a zone of formation of a thermal boundary layer, where temperature change occurs. Temperature distribution in the fluid flow core (which decreases along the growth of the dimensionless quantity of the pipe) remains almost homogeneous and almost equal to the temperature at the pipe inlet. A section of a pipe with all these characteristics got a name 'thermal inlet pipe section,' 'inlet pipe section,' or 'thermal stabilization pipe section.'

At some distance from the pipe inlet thermal boundary layers begin to merge, and heat transfer starts to take place at the whole cross-section of the pipe. Starting from a certain value of the dimensionless quantity of the pipe, fluid temperature profiles become similar, i.e. temperature at different pipe cross-sections differs only in absolute value, and the law of temperature change along the radius remains the same. In the first (inlet) section of the pipe, where fluid temperature profile is forming, Nusselt number decreases, and in the second section, where fluid temperature profiles become similar, Nusselt number stops changing.

For the purpose of determining heat transfer coefficient it is usually assumed that near the fluid surface (in the boundary layer) heat is transferred by thermal conduction. But we believe that this assumption may not be completely accurate, because Fourier's heat conduction law is only valid under condition of a constant heat conduction coefficient over the entire cross-section of the channel and constant temperature gradient [11]. However, usually fluid temperature gradient varies from zero at the channel axis to very high values at the channel wall, calling into question the common assumption described above.

Instead of heat transfer coefficient, Graetz and Nusselt introduced dimensionless Nusselt number into calculations. They showed that at $x \to 0$ Nusselt number tends to infinity: $Nu \to \infty$. Then the function rapidly decreases, and, starting from a certain value of the argument, and remains almost constant. At $x \to \infty$ $Nu \cong 3{,}66$. Based on this, they concluded that the maximum value of the heat transfer coefficient (at the length of the pipe tending to infinity) remains constant, and depends only on the fluid heat transfer coefficient λ and pipe's diameter d.

It seems that limiting Nusselt number that was obtained in solving this problem (non-zero value of the heat transfer coefficient with pipe length tending to infinity) is not consistent with the common assumption described above. If we assume that surface heat flux density is

$$q = \frac{dQ}{dtf},$$

then in a case of an infinitely long pipe $x \to \infty$, heat transfer surface area f must also tend to infinity: $f \to \infty$. Source power $\frac{dQ}{dt}$ determines wall temperature. If wall temperature does not change along the length of the pipe, power would also be constant. Correspondingly, if $f \to \infty$, surface heat flux density should tend to zero: $q \to 0$.

It follows from the expression (3) that the above is possible only when channel wall temperature and fluid temperature in channel are the same, i.e. in thermal equilibrium, or when heat transfer coefficient is zero. It also follows from the solution of the expression (3) that $\alpha \neq 0$. In thermal equilibrium the temperatures are the same, and temperature difference in expression (3) becomes zero, although heat transfer coefficient does not become zero. Then using heat transfer coefficient as a heat transfer characteristics stops making any sense, because it doesn't seem to play any role in heat transfer.

Thermal equilibrium is the most chaotic state of the system, and it can be expected to happen for sure in an infinitely long pipe. Then temperature across the pipe's cross-section will be the same as the pipe wall temperature, and self-similar flow established in the system should collapse as the length of the channel increases. After the inlet section and a section of thermal stability there should be another section – a transitional one, preceding the thermal equilibrium.

Then Graetz and Nusselt's assumption that fluid flow is a developed laminar flow along the whole flow area, from fluid entering the pipe to $x \to \infty$, would probably need clarifying. It follows from the mathematical formulation of the problem that at the inlet of the pipe fluid flow is laminar, and it remains laminar at any length of the pipe, including $x \to \infty$. This assumption would be unlikely reflecting reality even in the absence of heat transfer, because when laminar flow is formed, it only stays in the pipe section of a certain length due to the loss of flow stability. In the laminar (layered) flow impulse transfer in the crosswise direction to the wall is carried out only molecularly, since fluctuations arising in the boundary layer are damped by friction forces. When boundary layer (or channel length, or Reynolds number) grows, friction forces are not capable to damp flow disturbances anymore. Then a macroscopic transfer is added to the molecular impulse transfer to the wall, vortexes appear, and the flow becomes transient from laminar to turbulent.

Also, Graetz and Nusselt's assumption that heat transfer does not affect hydrodynamics of the flow becomes questionable as well. Finally, it becomes difficult not to question: is the expression (3) valid at all for determining heat transfer coefficient?

Petukhov and Case solved a problem of a fluid heated in a pipe under a condition of constant heat flow density ($q = const$) using analytical method [7,12]. They made simplifying assumptions, similar to the ones in the above mentioned problem with constant wall temperature. They assumed that flow and heat transfer are stable, fluid's physical properties are constant, fluid temperature at the pipe inlet is constant and homogeneous at a cross-section of the pipe, velocity profile in the entire flow is parabolic (laminar flow), and fluid's heat conduction in the in the axial direction does not affect heat transfer. Solution for this problem showed that, just like in the previous problem, there are two flow sections: thermal initial section and stabilized heat transfer section. In the first section Nusselt number tends from infinity at the pipe inlet to some asymptotic value at $x \to \infty$: $Nu \cong 4{,}36$. Looks like, heat transfer coefficient changes along the channel length from infinity at $x \to 0$ to some constant at $x \to \infty$. It is clear that this solution is not consistent with the principles of thermodynamics – in an infinitely long channel a thermal equilibrium should have been established between heated fluid and pipe walls. Also, this solution is inconsistent with the experiments [13].

Fig. 1 shows the curves of changes in heat transfer local coefficients along a pipe length: ordinate axis reflects heat transfer coefficient, abscissa axis shows energy surface density ($q\,x/v$)). As it follows from Fig. 1, at the inlet of the pipe (at $x \to 0$), heat transfer coefficients tend to infinity, and then decrease with the increase in pipe length. By looking at curves it's easy to conclude that each of them tends to some asymptotic value. Unfortunately, the length of the pipe under study was pretty small (a little more than one meter), and it was not possible to know whether heat transfer coefficients really have an asymptotic value, like Fig. 1 shows. However, it was possible to try to describe the trend line mathematically.

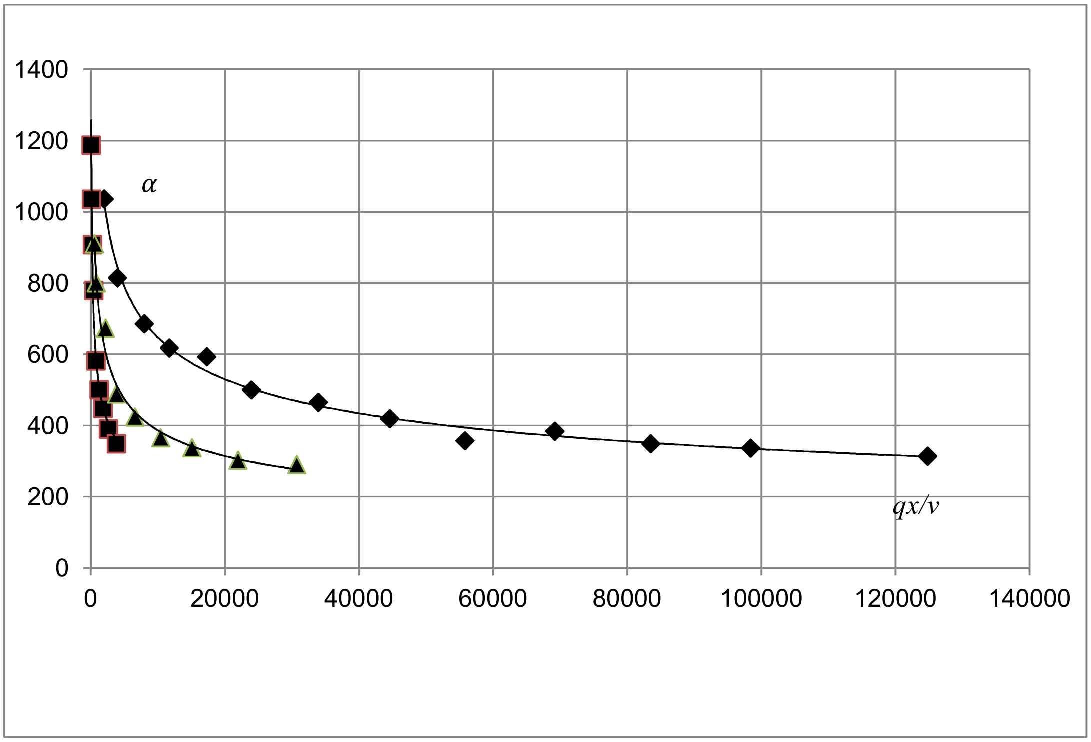

Fig. 1. Change of α along the channel length [13]

Using Microsoft Excel 2010, we decided that the closest to the experimental data trend line can be best described by an exponential function, similar to

$$\alpha = M_1(\frac{q\,x}{v})^{-0,3}$$

where M_1 is a constant. From this dependence it follows that if a pipe is infinitely long ($x \to \infty$), heat transfer coefficient tends to zero. According to the precise solutions, under these conditions heat transfer coefficient tends to being a constant, but according to experimental data, it tends to zero. Also, changes in wall temperature along heat transfer surface are not confirmed by experiments. And, according to precise solutions, in a stabilized flow wall temperature should change linearly with growing pipe length, but experimental curve looks more like logarithmic or exponential function.

3. ON NONLINEAR APPROXIMATION

It seems that in order to eliminate discrepancies between experimental data and theoretical solutions, linear system of equations (1-4) should be modified and presented as (5-8) [14]:

$$\frac{\partial \bar{v}}{\partial t} + \bar{v}\nabla v = -\frac{1}{\rho}\nabla P + \frac{1}{\rho}\frac{A}{(n+1)}\Delta \bar{v}^{n+1} \qquad (5)$$

$$div\bar{v} = 0 \qquad (6)$$

$$q = \frac{f}{\Pi}\rho C v \frac{d<T>}{dx} \qquad (7)$$

$$\frac{\partial T}{\partial t} + \bar{v}\nabla T = \frac{1}{\rho\,C}\cdot\frac{B}{n+1}\Delta T^{n+1} \qquad (8)$$

Here f – cross-section of a channel where fluid flows; Π – channel perimeter; $< T >$ – average temperature at a channel cross-section; A, B, n – some constants.

System of equations (5-8) allows calculating laminar transient and turbulent flows, and, at least qualitatively, correctly describes changes in velocity and temperature profiles during the transition from laminar to turbulent flow. If $n = 0$, Navier-Stokes and energy linear equations become a special case of nonlinear expressions (5) and (8).

Here we would like to introduce our approach to solving this problem. Our paper [2] shows why and how Navier-Stokes equation should be modified, and what should follow from it. Papers [9] and [11] reason why and how to use expression (7) instead of expression (3). Below we would like to show how to obtain a non-linear energy equation (8).

In deriving linear energy equation (4) it was assumed that heat is transferred from a channel wall to a moving fluid by thermal conduction (according to Fourier's law). Fourier's law is valid for small temperature gradients, provided that they are constant. When heat is transferred from the channel wall to the moving fluid, temperature gradient changes from zero on the channel axis to the very high values at the wall, and Fourier's law near the channel wall becomes invalid. Besides, the amount of energy received depends on the source temperature: the higher is the temperature, the more heat energy is transferred to the heat-receiving system. It means that the heat flow surface density q depends not only on the temperature gradient, but also on the numerical value of the temperature. It would seem that surface heat flux density equation should include both temperature and temperature gradient, because when describing something mathematically, functions are always represented by independent arguments only. But temperature and temperature gradient are not independent arguments. So, heat flow surface density dependence should be presented either as a function of temperature, or a function of temperature gradient. Note that Fourier's law

$$q = \lambda \nabla T$$

is valid when $\frac{\partial q}{\partial \nabla T} = \lambda = const.$

In order to take into consideration an effect of both temperature and temperature gradient on heat transfer, let's assume that a partial derivative of the heat flow density along the temperature gradient is not a constant value (as it is accepted in the linear theory), but represents an exponential function like (9), in which B and n are some constants:

$$\frac{\partial q}{\partial \nabla T} = B \nabla T^n \tag{9}$$

Using expression (9) for heat flux surface density, it was obtained in [11] that q is not a linear, but a power function (10):

$$q = \frac{B}{n+1} \nabla T^{n+1} \tag{10}$$

Theoretically, when deriving Navier-Stokes nonlinear equation, it follows that constant n can vary from -1 to zero. Value $n = -1$ corresponds to a well-developed turbulent flow. At $n = 0$ the flow is laminar (has parabolic velocity profile). It seems that during heat transfer n should change within the same limits too, and then the expression (10) would not contradict Fourier's law. When there's no flow velocity in medium and $n = 0$, expression (10) becomes indistinguishable from Fourier's law of heat conduction.

Now, let's obtain elementary volume dV in a shape of a fluid cylinder moving in a cylindrical pipe. According to the first law of thermodynamics, in order to change a temperature of a cylinder with a mass m by a value dT, it would be necessary to bring energy in the quantity of dQ to it. Mass m can be written out as ρdV. Then a change in the internal energy of the cylinder with a specific heat capacity C at constant volume will be

$$dU = \rho \cdot C \cdot dV \cdot dT$$

To find temperature change during dt, dT should be divided and multiplied by dt. Then the amount of heat, which must be brought up to the volume dV in time dt, should be equal to

$$dQ = \rho \cdot C \cdot \frac{dT}{dt} \cdot dV \cdot dt \tag{11}$$

If we divide expression (11) by dV and dt, we'll get expression (12):

$$\frac{dQ}{dV \cdot dt} = \rho \cdot C \cdot \frac{dT}{dt} \tag{12}$$

The amount of heat dQ, entering the allocated volume dV through its side surface S for the time dt, can be determined using an expression (13):

$$dQ = dq \cdot S \cdot dt \tag{13}$$

From the expression (13) it would be easy to find the value of $\frac{dQ}{dV \cdot dt}$:

$$\frac{dQ}{dV \cdot dt} = \frac{dq}{dV} S \tag{14}$$

Let's put q from the expression (10) in the expression (14), as well as the size of the cylinder's side surface and its volume, then we'll get the expression (15):

$$\frac{dQ}{dV \cdot dt} = \frac{B}{n+1} \cdot \Delta T^{n+1} \quad (15)$$

Then what remains is to equate the right parts of expressions (12) and (15), having preliminarily written out a complete (substantive) derivative of temperature. Then we'll get to the energy equation (8):

$$\frac{\partial T}{\partial t} + \bar{v}\nabla T = \frac{1}{\rho\, C} \cdot \frac{B}{n+1} \Delta T^{n+1} \quad (8)$$

Equation (8) differs from the usually used in calculations linear equation (4). It turns into well-known (4) only if $n = 0$. It follows from the expression (8) that temperature profile in channels changes from laminar to turbulent flow, and it becomes more filled. But it is not possible to draw these kinds of conclusions from linear energy equation (4).

An important advantage of using the system of equations (5-8) in comparison with (1-4) is the possibility to solve problems without using so-called "auxiliary quantity" – heat transfer coefficient. For example, if it is necessary to solve engineering problem of heating a fluid from temperature $< T_0 >$ to $< T >$ under a condition of a constant heat flow surface density, it can be done in a very simple way. For example, let's assume that the heating is done in a straight circular cylindrical pipe. In this case, we can write out expression (7) as an expression (16):

$$q = \frac{f}{\Pi} \rho C v \frac{d<T>}{dx} = \frac{r}{2} \rho C v \frac{d<T>}{dx} \quad (16)$$

If density, fluid heat capacity coefficient, and an average fluid velocity at a channel cross-section are constant and do not change during the heating process, the equation (16) can be easily integrated. But if thermophysical properties of the fluid depend on temperature, these temperature dependencies should be first entered into equation (16), and only then integrated.

Let's separate the variables and integrate equation (16) under boundary condition $x = 0$,

$< T > = < T_0 >$. Then we get a solution (17):

$$< T > - < T_0 > = \frac{4}{\rho \cdot C \cdot d} (\frac{q \cdot x}{v}) \quad (17)$$

Average fluid temperature along the channel changes linearly (which is confirmed by experimental data [13,14,15]), and depends not only on the surface heat flux density, but on the complex $\frac{q \cdot x}{v}$. This means that the same final temperature can be obtained by varying surface heat flux density, fluid velocity, diameter of the pipe, and length of the pipe. Note that the ratio $\frac{x}{v}$ can be interpreted as a time during which fluid stays in channel. And, as a whole, complex $\frac{q \cdot x}{v}$ at dimensionality [*J/m*^{2}] represents surface energy density (energy supplied to one square meter of heating surface).

4. CONCLUSION

Using a linear system of equations to depict convective heat transfer in channels does not allow properly describing the entire variety of experimentally observed facts. It seems that the reason for it may be the use of linear models (laws) in deriving Navier-Stokes equations and energy equations. Replacing linear equations with nonlinear, as well as use of a physically justified expression for surface heat flux density instead of expressions called 'cooling laws,' would allow solving wider range of problems, and their solutions would be better aligned with the experimental data.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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