*British Journal of Education, Society &
Behavioural Science*
2(3): 239-259, 2012

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Learning Geometry in a Large-Enrollment Class: Do Tangrams Help in Developing Students' Geometric Thinking?

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Authors' contributions

This work was carried out in collaboration between all authors. NMS designed the study, performed the statistical analysis, wrote the protocol, managed the literature searches and wrote the first draft of the manuscript. SA managed the analyses of the study. All authors read and approved the final manuscript.

Research Article

Received 4th June 2012
Accepted 17th July 2012
Online Ready 21st July 2012

ABSTRACT

Aims: 1. To investigate the students' levels of geometric thinking while engaging in in-class tangram activities in a large-enrollment class; 2. To find out the student's perception towards the use of tangrams in learning geometry.
Study Design: Case study research design.
Place and Duration of Study: The study took place at the University of Malaysia Sabah for a period of three hours.
Methodology: The sample consisted of 192 in-service primary school teachers (110 females and 82 males; age range 30-40 years). Student's degree of acquisition of van Hiele (1986) geometric thinking was measured using an in-class tangram activity worksheet and analysed using acquisition scales of Gutiérrez et al. (1991). Questionnaires with closed and open-ended questions were conducted to explore learner's insights and experiences of their learning.
Results: The results showed that a majority of student's attained complete acquisition of van Hiele level 1(88%) and level 2(82.3%). However, fewer than half (33.9%) of students could attain complete acquisition of level 3. These findings suggest that students

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developed through a sequence of hierarchical levels of thinking in learning geometry. Most of the students agreed that the tangram activities fostered their interest and appreciation towards geometry and boosted their confidence and creativity in learning geometry. The majority of them also agreed that tangram activities have enhanced their understanding of 2-D geometric concepts and the 3 levels of van Hiele model. All students hoped to implement this activity in their classrooms and promote it as recreation activities.
Conclusion: Tangram activities carried out in a large class helped in-service primary school teachers develop sequentially from the visualisation level through to the informal deduction level. Structuring learners' experiences using tangram and peer assessment not only served as an appropriate in-class exercise to facilitate students' geometric thinking, but also develop confidence, interest and appreciation toward geometry.

Keywords: In-class exercise; large-enrolment class; learning geometry; 3 levels of van Hiele geometric thought; tangram.

1. INTRODUCTION

Primary school geometry instruction seems to focus on the plane geometry, which is one of the Euclidean geometrical studies, investigating the properties of and the relationships between plane figures (Borowski and Borwein, 1989). Plane figures are two-dimensional geometrical shapes which are described by straight lines or curves (Nixon, 1887). Several examples of plane figures are circles, triangles, quadrilaterals and different types of polygons. In studying the properties of plane geometrical figures, students will identify and classify corners or angles, parallel sides, line or symmetry and other attributes. According to the New Jersey Mathematics Curriculum Framework (1995), the manipulation of concrete materials such as cut-and-paste constructions of paper models, formation of new shapes and simplification of shapes are considered useful exercises to aid learners in understanding geometrical shapes and their properties.

However, most content about geometry in primary school education has largely focused on knowing terms, definitions and attributes of shapes, while neglecting opportunities to manipulate concrete materials (Copley, 2000). Children tend to memorise mathematical facts such as names and attributes of shapes without understanding them. They learn geometry with misconceptions and carry these ideas throughout their adulthood. For example, many children are not aware that a square is a rectangle, some kites are squares and a square could be a rhombus, too (Connolly, n.d. and Kay, 1987). Lack of authentic experiences in the primary school, apart from inadequate teaching; have been identified as some of the possible reasons for students' misconceptions of geometry (Oberdorf and Taylor-Cox, 1999; Dickson et al., 1984). Teachers, thus, need to help children to develop and build up the right schemata. Learning activities should enable children to assimilate and accommodate experiences into the existing schemata.

Van Hiele (1986) proposes a five-level model describing how children learn geometry. These levels are product of experience and instruction, moving from visualisation, analysis, informal deduction, deduction to rigour. The van Hiele (1986) geometric model of thinking identifies shapes and figures according to their global appearances at the first ('visual') level for learners. At the second ('analysis') level, learners identify shapes according to their properties (for example, a rhombus is a figure with four equal sides). At the third ('informal deduction') level, learners are able to establish the relationships of properties both within figures (for example, in a quadrilateral, parallel opposite sides enables the angles to be equal) and among figures (for example, a square is considered a rectangle because it has all the properties of a rectangle). Thus, they can deduce properties of a figure and recognise classes of figures by simple logical deduction.

At the fourth ('deduction') level, students could produce a conclusion and understand that deduction is one of the methods in establishing geometric truth. At the fifth ('rigour') level, students could understand the formal aspects of deduction, such as establishing and comparing mathematical systems. The van Hiele theory further postulates that in learning geometry, learners seem to progress through a sequence of five hierarchical reasoning levels, from holistic and analytical thinking to rigorous mathematical deduction (Van Hiele, 1999:315; Van Hiele, 1986:39). Teachers, thus, should not cause students "to lose sight of the real relation between levels" (van Hiele, 1986, p. 53).

Children in the 6^{th} grade of primary school generally reach level 1 and start to move to level 2. At this stage, students should have gone beyond identifying basic geometric figures and analyse the properties of graphic, and learn to recognise the relationships between types of shapes (informal deduction level). Spear (1993) postulates that the first three levels are within the capacity of primary school learners. Thus, a primary school Mathematics teacher should at least attain the first three van Hiele's levels in order to guide his/her students effectively from one level of thinking to another.

The van Hiele's theory strongly emphasises the use of manipulatives in teaching geometry to facilitate the transition from one level to the next (Fuys et al., 1988). Manipulatives are defined as physical or concrete objects that are used as teaching tools to engage students in the hands-on learning of Mathematics (Boggan et al., 2010). When children touch and manipulate concrete objects, they become more proficient in knowing positions or locations in space (for examples: above, horizontal) and structure (for example: number of parallel sides). Ultimately, hands-on study of geometric objects helps young children develop a strong intuitive grasp of geometric properties and relationships (New Jersey Mathematics Curriculum Framework, 1995).

Singh (2006) found that tangrams are stimulating manipulative for children in order to acquire geometry thinking and reasoning process. A tangram is a dissected puzzle consisting of seven geometric pieces of shapes called *tans* (Bohning and Althouse, 1997). The seven pieces include a square, a parallelogram, two big right triangles, a medium sized right triangle and two small right triangles. The three basic shapes consist of a triangle, a square and a parallelogram, which fit together in various ways to form polygons such as a large square, rectangle, or triangle. They can also be arranged in a variety of complex shapes such as animals, birds, sea creatures, people and other figures (Bohning and Althouse, 1997).

Tangrams are an ideal manipulative in geometry teaching as it provides a concrete way to learn physical knowledge of Mathematics in order to understand geometric concepts (Bohning and Althouse, 1997). Tangrams allow children to develop geometric concepts by categorising, comparing and working out the puzzle and thereupon to solve problems in geometric contexts (Lin, 2011). Studies show that tangrams inspire children's observation, imagination, shape analysis, creativity and logical thinking (Olkun et al., 2005; Russell and Bologna, 1982; Yang and Chen, 2010). Consequently, playing with tangrams can help children to develop their skills of geometry vocabulary, shape identification, shape orientation and discover relationships between and among the 2-dimensional geometry shapes (Bohning and Althouse, 1997; Krieger, 1991; National Council of Teacher's Mathematics, 2003). Such experiences are especially important for young children to recognise and appreciate geometry in their natural world (Bohning and Althouse, 1997).

Unfortunately, proper usage of tangrams in Malaysian primary school classrooms is still lacking. A survey was conducted in reviewing current teaching practices among 192 primary school teachers throughout Sabah, East Malaysia. This survey reveals that 98 % of them have not used tangrams in teaching geometry, whereas 2 % of them used the tangrams pieces in isolation instead of asking students to form specific shapes with 7 tans. Without adequate instructions, primary school students would fail to use the tangrams in the intended manner and lose experiences that could sharpen their thinking skills, develop positive attitudes towards geometry, shape identification and orientation skills, and foster an understanding of basic geometric concepts and relationships. Thus, there is a need for primary school teachers to learn the proper way in using tangrams in the classroom in order to enhance student's geometric thinking. Teachers should provide a wide variety of geometrical experiences, especially the physical manipulation of shapes, so that students can gain a firm understanding of geometrical concepts.

The van Hiele levels place an emphasis on the importance of structuring learners' experiences to facilitate transition through the levels. Van Hiele (1986, p. 39), quoted from his 1955 source, states: "The attainment of the new level cannot be effected by teaching, but still, by a suitable choice of exercises the teacher can create a situation for the pupil favourable to the attainment of the higher level of thinking". Pegg and Davey (1991) believe that the van Hiele levels are basic knowledge to improve the teaching of geometry. Therefore, it is paramount for in-service teachers to possess an adequate understanding of van Hiele model and aware of their levels of understanding in geometry. This would allow them to teach the subject matter effectively by guiding their students through the levels themselves. Failure to do this would result, according to van Hiele, in rote learning in which students memorise the right answer without understanding. Hence, it is important to ascertain the in-service primary school teachers' level of van Hiele's thought in this study.

The deficiencies in teachers' level of van Hiele's thought can be attributed to the lack of effective teaching methods to promote the development of geometric thinking in their in-service training. Consequently, effective instructional methods should be employed in helping primary school teachers to attain the first three Van Hiele levels. What the teacher knows is one of the most important variables that could determine what will take place in the classroom (Turner-Bisset, 2001:148; Fennema and Franke, 1992:147). Thus, this data may lead to a reduction in the current apprehension about geometry in students as proven in the latest Trends in International Mathematics and Science Study (TIMSS, 2007) Report. The average geometry achievement of Malaysian Form Two students was not only significantly lower than the TIMSS scale average but also even further below the average geometry achievement of the top five Asia-Pacific countries of Chinese Taipei, Republic of Korea, Singapore, Japan and Hong Kong (Mullis et al., 2008). This indirectly reflects that our Malaysian Form Two students' levels of geometric thinking are still far from satisfactory. In addressing this concern, primary school students ought to be equipped with solid foundation of geometric knowledge before they continue their education in secondary schools.

In response to the current development on geometry among secondary school students, the revised Malaysian Curriculum specifications of integrated curriculum for primary schools

Mathematics (Malaysian Ministry of Education, 2006) include ‘Shape and Space’ as one of the four areas of Mathematics at the lower and upper primary levels (along with numbers, measures, and statistics). From this specification, it can be concluded that geometry has constituted as much as one fourth of the primary Mathematics curriculum. The success of any revised Mathematics curriculum that gives greater prominence to geometry would inevitably depend, to a large extent, on the content knowledge of the teachers who teach it and the teaching methods employed. Therefore, it is crucial for primary school teachers to receive professional development within the profession to keep in touch with new findings in their subjects and to obtain continuous support for the improvement of their teaching methods.

The urgent need for professional development opportunities for teachers in the area of teaching and learning is stressed by the Tenth Malaysian plan (2011-2015), that the Government undertakes measures to upgrade the quality and professionalism among all existing teachers. The proportion of graduate teachers in primary schools will be increased from 28 per cent in 2009 to 60 per cent by 2015 (Malaysian Economic Planning Unit, 2010). To achieve this goal, various programmes enabling non-graduate teachers to attain degrees will be intensified. Meanwhile, the number of student intake will be increased up to 200 students per programme. For example, for May 2011 intake, a university in Sabah, Malaysia, had accepted 192 primary school teachers per cohort into its Bachelor of Education (Primary Mathematics Education) programmes.

Due to high enrollments in graduate programmes for primary school, it appears that university administrators considered large classes as economically advantageous. Previous studies revealed that large classes could limit teachers’ abilities to implement discussion, timely feedback and active problem solving. All these activities seem to be connected to success in secondary and undergraduate education, as well as short- and long term achievement growth among students (Kulik and Kulik, 1989; Nye et al., 2002). Furthermore, large classes could overburden teachers and limit teacher-students interaction (Longmore et al., 1996). This situation could lead to negative aspects of teaching and learning, which usually implicates teachers and students themselves.

However, recent literature reviews (McKeachie, 2002; Toth and Montagna, 2002) suggest that class size does not appear to negatively impact achievement of low-level learning objectives. Research has shown that class size does not automatically correlate with student learning (Felder, 1997). Students in large classes can learn just as active as those in small ones (Bouton and Garth, 1983). What counts is not the size of the class, but the teaching quality. Still in the same vein, Haddad (2006) suggests that certain characteristics are necessary for effective teaching in large classes. These include creating a well-managed classroom community so that students are ready to learn in a comfortable physical and psycho-social environment; choosing effective alternatives to the standard lecture format; and evaluating learning and teaching in large classes, so that students could show what they have learned, and teachers can reflect on their own teaching practices, as well.

Nevertheless, the task of ensuring manageable assessment is much more difficult with large classes because of the marking load involved (McKeachie, 1999; Stefani, 1998). Gibbs (1992) argues that assessment in large classes is manageable without undermining the quality of learning. One of the suggested strategies is using peer-assessment as part of a learning experience. Hanrahan and Isaacs (2001) share the same view that peer-assessment can provide useful learning experiences for students, while reducing the marking load of teachers. They also mention that peer assessment allows students to view the work of their peers and contemplate the merits of other ways of approaching the assessment, as well as the importance of adherence to the marking criteria. It is particularly advantageous for students to provide their feedbacks, as this ensures that they could carefully consider the merits and deficiencies of their works.

The concern of the researcher then, who is a Mathematics educator, dealing with a large-enrollment class for primary Mathematics education, is how to make learning environment in large classes more meaningful. Issues such as how to prepare and plan a lesson specifically adapted for a large class were addressed. The researcher wanted to change the perspective from teaching that focuses solely on didactic approaches to more student-centred and learning-friendly methods. Felder (1992) advocates the use of in-class exercises to provide a plenty of active learning experiences in a large class instead of relying on straight lecturing. Thus, this study addressed this concern by integrating student-centred methods within-class tangram activities and peer assessment in the teaching and learning of geometry in large classes.

1.1 Objective of the Study

This study attempts to investigate whether the use of tangram as an in-class activity following the van Hiele's geometric model could help students in a large-enrollment class to develop geometrical thinking skills. In this study, geometrical thinking skills refer to van Hiele's three-level hierarchy of thinking processes used in exploring geometric concepts: visualisation, analysis, and informal deduction. Specifically, geometrical thinking constitutes skills such as constructing, drawing and naming additional geometric shapes which are constructed from the given tans (visualisation), classifying of shapes according to properties and deriving generalisations inductively (analysis), as well as interrelating several classes of shapes using Venn diagram (informal deduction).This study, thus, attempts to show how primary Mathematics teachers can maximise the use of tangrams, and determine how the seven tans of tangram can be manipulated by children in order to gain geometrical thinking.This study embarks on the following objectives:

1) To investigate the students' levels of geometric thinking while engaging in tangram activities in a large-enrollment class;
2) To find out the students' perception towards the use of tangrams in learning geometry.

More specifically, this study addresses the following questions:

1) Which level of geometric thinking do students attain by the end of tangram activities carried out in a large-enrollment class?
2) What are the students' insights and experiences about using tangrams in learning geometry?

2. RESEARCH METHODOLOGY

The study took place in University Malaysia Sabah. In this study, the researcher followed the "convenience" sampling procedure defined by McMillan (2000), where a group of 192 in-service primary school teachers throughout Sabah and Sarawak, Malaysia was selected because of its availability. They were 110(57.3%) females and 82(42.7%) males. They were pursuing a Bachelor of Education, majoring in Primary Mathematics. The age of the participants ranged from 30-40 years old. They have about 10-20 years teaching experience in primary schools.1% of the teachers involved have never taught Mathematics in their schools. 6.8% of them indicated that they have taught Mathematics in schools, and they were neither a major nor a minor when doing their teaching diploma programme. 11% of them were teaching in Chinese medium primary schools whereas 89% came from Malay medium primary schools. The in-service primary school teachers were required to attend a geometry course, known as Basic Shape and Space, for one semester during weekends. From the demographic part of the questionnaire, the researcher has learned that 98% of them have not used tangrams in teaching geometry, whereas 2% of them have used the tangrams pieces in isolation instead of asking students to form specific shapes with 7 tans.

The researcher has developed two tangram lessons based on relevant educational literature on the subject (Umar, 1994) and van Hiele (1986) levels of geometrical thinking. The questionnaire's items were modified from the studies conducted by Lin (2011) and Abdullah and Zakaria (2011). The items follow the 5-Likert scale options, having 1=strongly disagree (SD), 2=disagree (D), 3=unsure (U), 4=agree (A) and 5=strongly agree (SA). Students were also asked to provide some written feedbacks on the activities for the open questions which include, "What did you learn from the activity using Tangrams?"; "What was your experience?" and "What were your thoughts after participating in the tangram activities?". Two experts comprising a Mathematics lecturer and a Malay language native speaker reviewed the Malay version of the questionnaire and tangram lessons in terms of their language and content.

A pilot study was conducted in the form of a 3-hour workshop to refine the questionnaire and tangram lessons. A total of 30 Year Two undergraduate Mathematics students took part in the pilot study. Prior to the pilot study, the researchers briefed the students involved in two sessions. They were taught how to create, explore, and manipulate the 7 tans of tangram and conducted activities as planned for the field study. Once the workshop ended, the students were asked to complete a questionnaire asking their perception regarding the conducted activities. Students were also asked to comment and make suggestions regarding the questionnaire and tangram lessons. The students agreed that all items were relevant, and they should remain in the final questionnaire and tangram lessons. The index of reliability of the questionnaire is 0.96.

2.1 The Learning Process

The Basic Shape and Space course was taught to 192 students in a large lecture hall. In creating a comfortable learning environment to maximise learning, students sat at their own tables to carry out tangram activities. The tables were arranged side by side so that students could seek for help from peers when they encountered difficulties. Student's degree of acquisition of each van Hiele's level was measured using a tangram activity worksheet. The lesson lasted for three hours and the process was as follows:

2.1.1 Part 1 (1 hour) – constructing a tangram set and investigating shapes

1. Introduction: Folding and cutting apart the seven tans
   To get the students' attention at the beginning of the lesson, a video clip was used to demonstrate the creation of a tangram puzzle by using a square piece of construction paper. The students folded and then cut the seven pieces (called tans) apart. Each student has his or her own set of tangram pieces to be used in the activities. He/She was introduced to the intended rules for creating tangram figures – all pieces must touch and no piece can overlap.

2. Investigating the seven tans
   Active learning strategies were employed, involving students doing their own activity and thinking about the seven tans. Students were given ample time to freely explore the shapes' similarities and differences, discover the shapes' attributes and relationships, and manipulate the pieces in a variety of arrangements. They would soon find multiple solutions for assembling a shape.

2.1.2 Part 2 (2 hours): in-class tangram activities

The following tangram activities illustrate the sequence of task for developing geometric thinking at the visual level and for supporting a transition to the informal deduction level.

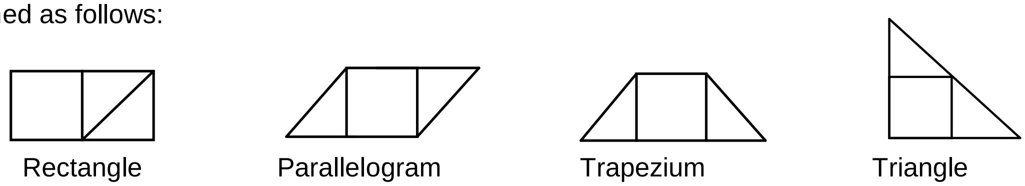
2.1.2.1 Task 1: Visualisation

The challenging part of this activity is to arrange the pieces to form additional/complex shapes. Students have to form different shapes from the pieces given. There were four activities and the instructions given were as follows:

What shape(s) will you form from the following tans?
1. A square and two small triangles.
2. Two small triangles and one parallelogram.
3. Two small triangles, one parallelogram, and a medium triangle.
4. Two small triangles, one parallelogram, and a medium triangle and a square.

For each task, students have to explore and manipulate the given geometric shapes (tans), and construct by drawing and naming according to their global appearance. They were also asked to answer a series of prompting questions leading to thinking-based learning such as "How do the given tans work together to make another polygon?," "What is the shape of each piece of polygon formed?," "What is the common characteristics of each group," and "How can the relationships between and among the geometry shapes or polygons be represented in a Venn diagram?". This process could enable primary school teachers to get a more accurate picture of how the tangram works in a classroom.

For example: The tans given are one square and two small triangles. Shapes could be formed as follows:

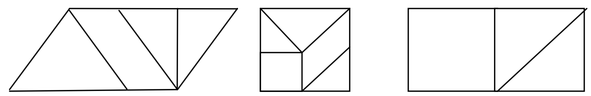


2.1.2.2 Task 2: Analysis

Students had to recognise the properties of the shapes and classified them according to their properties, and derived rules of a class of shapes inductively by analysing figures in terms of their components and relationships among them. The students operated with these rules both within a class of shapes and between related classes of shapes. Students also listed down all the common characteristics belonging to the group.

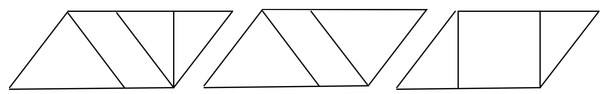
Between related class of shapes

Students investigated the relationships among squares, rectangles and parallelograms. For example: Opposite sides are parallel and congruent and opposite angles are congruent. Thus, squares and rectangles are parallelograms.



Within a class of shapes

For example: Opposite sides are parallel and equal, and opposite angles are equal. Thus, all these shapes are parallelograms.



2.1.2.3 Task 3: Informal Deduction

This was followed by a discussion on the thinking tools that helped students to enhance their thinking abilities. Students have to relate several properties found in the geometric shapes, justifying the conclusion using logical relations and representing it in a Venn diagram. They may summarise the properties of each type of quadrilateral and relate them using a Venn diagram. For example: A square is a special form of rectangle and a parallelogram that has four right angles and four equal/parallel sides in length. A rectangle is a parallelogram and a quadrilateral that has two pairs of parallel sides and four right angles. A parallelogram is a quadrilateral that has two pairs of parallel sides. Quadrilaterals are shapes with four sides and four angles. Thus, a square, rectangle and parallelogram can be classified as quadrilaterals (Fig. 1).

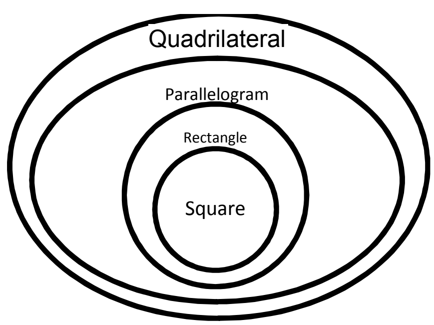


Fig. 1. Venn diagram

During the lessons, the trainer monitored the activities of all the students and provided assistance, where necessary. Before the lesson ended, a recreation activity was carried out when students were asked to assemble the seven tans in many different ways to make figures of birds, animals, people, and other objects. Students then shared their discoveries with their peers.

2.1.3 Peer assessment

To make sure everyone worked on the task in an in-class tangram exercise and learned from it, student' worksheets could be amended for grading. Students were warned that the submitted answer sheets would undergo a similarity check. Should plagiarism be detected, zero mark would be given. Peer evaluations were carried out in which every student was involved to help with the grading process. In this regard, peer-assessment could provide students with a positive learning experience by compelling them to examine their work from the perspectives of a marker as well as a learner.

The marking scheme was prepared in advance using power point slides. Once all of the students have finished, they exchanged their worksheets and referred to the marking scheme to grade their partner's worksheets. To make sure the markers took the marking seriously, the students were reminded to write down their names at the bottom of the answer sheets and marks would be given to the markers based on the accuracy of their markings. By using this strategy, the students can find out how they have performed and what they need to do in order to improve. The researcher then collected the worksheets and carried out cross-marking.

The distribution of questionnaires was administered promptly at the end of the intervention, which lasted for 15 minutes. These questionnaires with closed and open-ended questions were conducted to explore learner's insights and experiences in their learning. Student's answer sheets were collected and evaluated to determine the van Hiele's levels of geometric thinking.

2.2 Data Analysis

2.2.1 Analysis of students' level of thinking

In order to assign students to a specific degree of acquisition within each van Hiele's level, the researcher followed the methods proposed by Gutiérrez et al. (1991). First, each student's answer was classified according to the task descriptors for the van Hiele's levels 1 to 3 (Table 1) which were developed based on van Hiele's level descriptors developed by Han (2006). Then, depending on its geometric accuracy and on how complete the solution to the tangram activity was, each answer was assigned to one of the following types and its numerical weight as shown in Table 2. Answers proving two consecutive levels were assigned to the higher level because they indicate a certain degree of acquisition of that level.

The weightings from the "*Degrees of Acquisition of a van Hiele Level Scale*" were assigned by numerical scales graduated from 0 to 100 as shown in Table 3 below. The eight types of answers indicate the various degrees of acquisition of the van Hiele's levels. Types 0 and 1 indicate no acquisition, Types 2 and 3 indicate low acquisition, Types 4 indicates intermediate acquisition, Types 5 and 6 indicate high acquisition and type 7 indicates complete acquisition" (Gutiérrez et al., 1991, p.241).

Table 1. Task descriptors for the van Hiele's levels 1 to 3

Task and descriptions
1. Visualisation Students identify 2-D shapes (e.g. triangle, rectangle, square, parallelograms, and trapezoids) and other geometric components (e.g. angles, parallel lines) according to their global appearance. Students can construct, draw and name correctly as many as their created 2-D shapes from the tans given. They distinguish the created shape from others on a visual basis. The students do not need to explicitly list down or describe the properties/attributes of the shapes.
2. Analysis The students analyse 2-D shapes in terms of their components (sides, edges, angles, etc.), and relationships between components, establish properties (parallelism, regularity, congruency, etc.) of a class of 2-D shapes and generalise properties for that class of 2-D shapes inductively. Students are able to list down all the shared properties for a class of 2-D shapes within family or between families.
3. Informal deduction The students are able to logically classify 2-D shapes and recognise subclass relationships between different types of 2-D shapes and interrelate them using a Venn diagram.

Table 2. Descriptions and weights of each type of answer (Gutiérrez et al., 1991, p.240)

Answer type and Weight		**Descriptions**
0	0	No reply or answers that cannot be categorised.
1	0	Answers that indicate that the learner has not attained a given level but that give no information about any lower level.
2	20	Wrong and insufficiently worked out answers that give some indication of a given level of reasoning: answers that contain incorrect and reduced explanations, reasoning processes or results.
3	25	Correct but insufficiently worked out answers that give some indication of a given level of reasoning: answers that contain very few explanations, inchoate reasoning processes, or very incomplete results.
4	50	Correct or incorrect answers that clearly reflect characteristics of two consecutive van Hiele's levels and that contain clear reasoning processes and sufficient justifications.
5	75	Answers that represent reasoning processes that are complete but incorrect or answers that reflect correct reasoning but that still do not lead to the solution of the stated problem.
6	80	Correct answers that clearly reflect a given level of reasoning but that are incomplete or insufficiently justified.
7	100	Correct, complete, and sufficiently justified answers that clearly reflect a given level of reasoning.

The acquisition level obtained by the students was determined by quantifying the vectors (level, type) corresponding to all the tasks which could have been answered at that level (Gutiérrez et al., 1991, p.245). Table 4 shows the vectors (level, type) by a student. Table 5 shows that quantified weights assigned to each task.

Task 2 was answered at level 2 and Type 6, which implies complete acquisition of level 1 (weight=100), and high acquisition of level 2 (weight = 80). Since task 3 could have been answered at Levels 2 and 3, thus the fact that it had been answered in the example at Level 3 and Type 4 implies complete acquisition of level 2 (weight=100), and intermediate acquisition of Level 3 (weight = 50).

Table 3. Degrees of Acquisition of a van Hiele's Level Scale

Degrees of acquisition and weight		Description
No acquisition	0 – 15	Learners are not conscious of the existence of thinking specific to a new level. They have no acquisition of this level of reasoning.
Low degree of acquisition	16 – 40	Learners are aware of methods of thinking, know their importance and try to use them. These learners make some attempts to work on this level, but have little or no success due to their lack of experience.
Intermediate degree of acquisition	41– 60	Learners use methods of the higher level more often and with increasing accuracy but still fall back on methods of a previous level. Typical reasoning is marked by frequent jumps between the two levels.
High degree of acquisition	61– 85	Characterised by progressively strengthened reasoning that indicates a learner is using a higher level of reasoning. Learners still make some mistakes or sometimes go back to the lower level.
Complete acquisition	86–100	Learners have completely mastered the new level of thinking and use it without difficulties.

Table 4. Vectors of a student's responses to task

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	Task		
	1.Visualisation	**2.Analysis**	**3. Informal Deduction**
van Hiele Level *(l)*	1	2	3
Type*(t)*	7	6	4

Table 5. Weights of the student's responses

van Hiele Level *(l)*	Tasks			
	1. Visualisation	**2. Analysis**	**3. Informal Deduction**	**Average**
1	100	100	-	100
2	0	80	100	60
3	-	-	50	50

Each student's degree of acquisition of each van Hiele's level was determined by averaging the values assigned to tasks that measure each particular level. The student in the example was assigned, having a complete acquisition of level 1 (average=100), intermediate acquisition of Level 2 (average=60) and intermediate acquisition of Level 3 (average=50).

2.2.1.1 Inter-rater reliability

The student's answer sheet was analysed and scored according to the descriptions and classification of the degrees of acquisition of van Hiele's levels by Gutiérrez et al. (1991). To determine inter-rater reliability, the classification of the degrees of acquisition was carried out independently by the researcher and a college Mathematics lecturer, both of whom have extensive background in primary school Mathematics education. In cases of disagreement on the assigned score of an answer, a discussion would be held to agree on a single score. The degree of agreement between the two independent classifications was calculated by Cohen's Kappa analysis (Agresti, 1990), using Crosstabs technique in the SPSS program version 17.0 for Windows. A summary of the crosstabs results for measuring agreement among the raters is presented in Table 6. The obtained Kappa is greater than 0.70, which indicates that the inter-rater reliability is satisfactory.

Table 6. Summary of Kappa values

	Van Hiele's Level (n= 20)		
Inter-rater reliability	Level 1	Level 2	Level 3
Kappa	0.829	0.771	0.772

2.2.2 Analysis of questionnaire data

Data was collected from 192 students about their perception towards the use of tangrams in geometry learning. The questionnaire was analysed according to Likert's five-point scale ranging from strongly disagree (1) to strongly agree (5), with a 3 representing a neutral response. Student questionnaire's items and percentage as well as mean scores of responses on the five Likert-type items are summarised in Table 8. The qualitative data consisted of students' responses from open-ended questions that investigated their insights and experiences in learning by using tangrams. All qualitative data were analysed by an iteractive coding process (Emerson, Fretzand Shaw, 1995). The analysis began by reading each of the participant's responses to survey open-ended questions. Codes were generated during this initial review of the participants' responses. Relationships among the codes were explored in subsequent readings of participants' responses and broad perspectives emerged. This process continued until consistent perspectives were achieved.

3. RESEARCH FINDINGS

3.1 Participants' Degrees of Acquisition at Each van Hiele's Level

Table 7 below summarises various degrees of acquisition of the van Hiele's levels of thinking attained by the students. From the table, it can be seen that, the higher the level, the lower the percentage of students who obtained complete degrees of acquisition. As shown in Table 7, 88% of the students attained a complete acquisition of van Hiele Level 1, and 82.3% of them attained a complete acquisition of the Level 2. Nevertheless, only 33.9 % of students attained a complete acquisition of van Hiele Level 3, 42.7% attained a high acquisition of Level 3 and the remaining attained intermediate (11.5%), low acquisition (8.3%) and no acquisition (3.6%) of the level.

Table 7. Number and percentage of students attaining degrees of acquisition of each van Hiele's Level

Van Hiele's Level	Degree of acquisition (n = 192)				
	No acquisition	Low	Intermediate	High	Complete
1			1(0.5%)	22(11.5%)	169(88%)
2			14(7.3%)	20(10.4%)	158(82.3%)
3	7(3.6%)	16(8.3%)	22(11.5%)	82(42.7%)	65(33.9%)

3.2 Participants' Perceptions on Tangram Activities

The results shown in Table 8 are based on the students' perceptions on the conducted tangram activities. For the purpose of discussion, Strongly Disagree (SD) and Disagree (D) are stated as "disagree", Agree (A) and Strongly Agree (SA) as "agree", while "unsure" is maintained. Students' perceptions and experiences about tangram puzzle activities were analysed according to the five facets of the activities: Usability, Strategies in Problem Solving, Motivation and interest, and Learning Gains.

The finding shows that a majority of students agreed that the 7 tans tangram were easy to be created (96.9% agreed) and manipulated to form additional 2-D shapes (83.3% agreed). Students agreed that they had applied strategies such as self-exploratory (92.2%) in trying new patterns without waiting for instruction, and peer help (94.8%) when they encountered difficulties. A majority of the students agreed that the conducted activities increased their interest towards geometry (99.5% agreed), attracted them to learn geometric topics (100% agreed), encouraged them to use tangram activities in their future teaching (98.4% agreed), encouraged them to participate actively in learning geometry (97.4% agreed), and made them to expect more tangram activities in Mathematics class(99% agreed).In terms of learning gains, a majority of the students agreed that tangram puzzle activities helped them to enhance their understanding about 2-D geometric concepts (96.8% agreed) and 3 levels of Van Hiele Model (96.9%),derive relationship between several classes of shapes (97.9%), think creatively (99.5%), foster appreciation towards geometry (99%) and gain confidence to solve geometric problems (99.5% agreed). Moreover, 100% of the students agreed that tangram activities can be promoted as good recreation activities in class.

3.3 Findings from the open-ended questions

Individual responses from open questions: "What did you learn from the activity using Tangram?;" "What was your experience?;" and "What were your thoughts after participating in the tangram activities?".

The analyses of students' feedbacks in responding to the open-ended questions seem to support the close items. The responses from open-ended questions were analysed to investigate the learners' insights and experiences in tangram activities as well as the process of learning. The written responses are analysed from four perspectives: (1) direct feelings towards the activities, (2) perceived learning outcomes in geometry learning, (3) perceived learning outcomes in creativity development, and (4) perceived expectation in implementing the tangram activities. The validity of open-ended response was determined by agreement between a Mathematics lecturer as independent rater and the researcher.

Table 8. Student questionnaire's Items, percentage and mean scores on each likert type item (n = 192)

Item	Percentage of students (%)			Mean
	Disagree	Unsure	Agree	
Usability				
2. It was easy to create 7 tans based on instruction given in the video clip		3.1	96.9	4.58
16. I didn't face any problem to produce additional/complex 2D shapes from the tans given	2.6	14.1	83.3	4.03
Strategies in problem solving				
3. I would ask for help from my partners if I encountered difficulties.	0.5	4.7	94.8	4.22
4. I would keep on trying new patterns without waiting for instruction	1.0	6.8	92.2	4.22
Motivation and interest				
5. Tangram activities have promoted my interest towards geometry		0.5	99.5	4.68
8. I would like to have more tangram activities in future geometry class		1.0	99	4.64
10. I look forward to using tangram activities with my students		1.6	98.4	4.55
11. Tangram activities have encouraged me to participate actively in learning geometry		2.6	97.4	4.66
13. Tangram activities have attracted me to learn geometric topics			100	4.63
Learning gains				
1. Tangram helped me to improve my understanding about 2D geometric concepts		3.2	96.8	4.58
6. I gain confidence to solve geometric problems		0.5	95.5	4.68
7. Tangram helped me to derive relationship between several classes of shapes		2.1	97.9	4.42
9. Tangram helped me to gain better understanding about 3 levels of van Hiele Model		3.1	96.9	4.67
12. Tangram helped me to think creatively		0.5	99.5	4.68
14. Tangram activities can be promoted as good recreation activities in class			100	4.75
15. Tangram fostered my appreciation towards geometry		1.0	99	4.62

Concerning direct feelings towards tangram activities, a majority of the students felt that the tangram activities were enjoyable, interesting, and motivating. Their feedbacks are:

> *"Very interesting, I want more mathematics activities with tangram puzzle;"*
> "*A fantastic activity, from a piece of square paper I am able to form 7 tans with different sizes and shapes. When those pieces were reassembled, it will form another polygon;"*
> *"I enjoyed learning geometry using tangram puzzle;"*
> *"An interesting way to learn geometry that used to be quite boring;"*
> *"I gain motivation in exploring geometry. There are many shapes can be assembled from the given pieces."*

Another direct feeling towards the activities is flexibility in learning. They mentioned that:

> *"I can explore and create more/new shapes freely using the same tans;"*
> *"By using the tangram, I am able to form different shapes easily;"*
> *"It is so amazing that the seven pieces can be arranged to produce so many different shapes."*

The tangram activities have also fostered their cooperation in solving problems. As students said:

> *"I was engaged to compare my work (tangram puzzle) with my classmates and discuss how to solve the problems."*

Students generally learned from peer assessment and felt 'ownership' of their outcomes. As students remarked:

> *"Since this activity will be marked by our friends, I have tried my best to produce the new shapes from the pieces. Even though it is very challenging for me, I am satisfied from my efforts to arrange and draw the shapes using my own creativity;"*
> *"I learn something when I mark my friend's worksheets. I was able to see other shapes which are different from mine, so actually we learn from each other."*

Concerning perceived learning outcomes in creativity development, students felt that by playing and manipulating each piece of the tangram puzzle, the activities help them to enhance creativity and imagination. Related responses are as follows:

> *"I gained a better understanding of geometric shapes after rotating and moving each piece of the tangram puzzle. It enhanced my mind to get more shapes;"*
> *"It helped to stimulate my imagination by creating new geometric shapes."*

Concerning perceived learning outcomes in geometry learning, nearly all the students felt that they have experienced the three levels of Van Hiele Model and therefore, their skills in describing and interrelating properties of groups in geometric shapes were enhanced. As they said:

> *"I learn a new way of relating different shapes using Venn diagram;"*
> *"The activity allows me to explore and describe the properties of 2-D shape in detail;"*
> *"It is easy for me to identify the properties of geometric shapes with the help of tangram puzzle;"*
> *"The activities help me to compare contrast shapes easily;"*
> *"The tangram activities help to consolidate ideas and further my understanding of geometrical concept."*

Another perceived learning outcome was that appreciation towards geometry was enhanced. As they mentioned:

> *"I began to appreciate geometry shapes in my surrounding;"*
> *"The activities eliminate my previous feelings that geometry is a tough and boring subject;"*
> *"I love to explore the field of geometry."*

Concerning perceived expectation in implementing the suggested activities, many students expressed that they hope to use tangram activities into their teaching. They wrote,

> "*A new approach in learning geometry that is worth to be implemented in schools;*"
> *"Will use this tangram as a recreation activity in class that helps develop pupil's creativity;"*
> *"I gain new experience to teach geometry to my students in future."*

4. DISCUSSION

The results from this study found that a majority of the primary school teachers had completely mastered the visualisation and analysis level of Van Hiele and used it without difficulties. Nevertheless, fewer than half of primary school teachers could completely master the informal deductive level of van Hiele. In other words, they were operating at the informal deduction level, but still made some mistakes or sometimes went back to the analysis level. The results further support the finding of previous research that students' geometric thinking is hierarchical in nature. Students progressed sequentially through the levels of van Hiele's geometric thinking in the process of learning geometry (Crowley, 1987; Hieles, 1986). This means that for the student to attain the geometric thinking at van Hiele level 3, he or she should completely master the van Hiele levels 1 and 2 of geometric thought.

Students taught according to the Van Hiele model with the help of tangram activities had to explore, try, and manipulate the 7 tans of tangram. The learner solved problems that require him or her to visualise and think geometric shapes – draw such shapes, discover their properties, make generalisations pertaining to such shapes, and form the relationship among the group of shapes. The open exploratory activities have increased their ability to think about geometry. In the process of learning, particularly in using tangrams as manipulatives and making the students think, they were oriented to logically interrelate previously discovered properties and rules using the Venn diagram, which fostered the development of their geometric thinking from visualisation to informal deductions level. It is from the exploration and manipulation of tangram puzzle activities that these students' understanding of 2-D geometric concepts and 3 levels of Van Hiele's Model was further enhanced. This is in line with the finding of Singh (2006), who found that tangrams are stimulating manipulatives for learners to acquire geometry thinking and reasoning process. Research has also documented that tangram experiences help learners to promote their shape identification and classification skills, apart from fostering an understanding of 2-dimensional geometric shapes and relationships (Bohning and Althouse, 1997; Krieger, 1991; National Council of Teacher's Mathematics, 2003).

Tangram provides students more opportunities to freely explore the 7 tans through a variety of activities, and work independently in a large class. Students sought help from peers when they encountered difficulties. As a trainer, the researcher then had time to focus her attention on facilitating and furthering the students' learning. This instruction takes the student-centred approaches that is learning by doing, and experience as the basis. The grading of tangram activities had given students a sense of ownership of the outcome, thus serves as an appropriate in-class exercise to facilitate students' geometric thinking in large class. This is supported by Felder (1992) who advocates the use of in-class exercise to provide plenty of active learning experience in large class, instead of relying on straight lecturing.

The students' feedbacks about the tangram activities were very positive. They found that the 7 tans of tangram were easy to be created and manipulated to form additional 2-D shapes, thus gaining confidence to solve geometric problems. Students were motivated to create, explore, manipulate and draw their own tangrams. In the process, they have developed high interest and appreciation towards geometry. This is in line with Bohning and Althouse (1997) finding which claims that tangram experiences help learners to develop positive attitudes towards geometry. Stipek (1998) also notes that the study environment and tasks given could provide a deep impact to the students' attitude.

Manipulating the seven pieces to form additional shapes fostered creativity in students. Students saw connections among shapes, and explored geometry with less anxiety. By just playing with the seven pieces, the students discovered the secrets inherent in the ingenious seven piece tans. As a consequence, students saw the relevance of tangram activities to foster creativity in primary school students in classroom.

5. CONCLUSION

Tangram activities which were carried out in a large class helped in-service primary school teachers develop sequentially from the visualisation level to the informal deduction level of thought. The primary school teachers who participated in the study were operating at the visualisation and analysis levels, but lacked informal deduction level. But still, by using tangram as in-class exercises in large class, an increased learning environment has been created for the students' inclination to the attainment of level three of van Hiele's geometric thinking. This could suggest that in-service primary school teachers need more attention in their in-service training of the kind of geometry thinking that Euclid created, which involves formal deduction. Primary school teachers need to acquire these forms of geometric thinking in order to forge connections in their knowledge of geometry, students and pedagogy, in ways that could enable them to help students learn effectively.

The results of this study suggest that the structuring of learners' experiences using tangram and peer assessment had served as an appropriate in-class exercise to facilitate students' geometric thinking. Tangram has provided students with increased opportunities to take responsibility for hands on and minds on learning activities that occurred within a large classroom. By creating, exploring and manipulating tangram puzzles, students found the joy and motivation of learning geometry. Tangram experiences have not only helped primary school teachers to enhance their understanding about 2-D geometric concepts and 3 levels of Van Hiele Model, but developed their levels of confidence, interest and appreciation toward geometry, as well.

This study provides suggestions on how primary school teachers can structure tangram activities to encourage the development of geometric thinking among their students. If Malaysian students are to be at par with students of other countries in geometry achievement in TIMSS, then primary school students should be stimulated to develop their visualisation and use of the second and third levels of Van Hiele's thought.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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