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Bayesian Analysis of Poverty Rates in the South-Western Part of Nigeria

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Poverty is global serious issue which differs in various cultures across the world and over time, varies according to the understanding of the society. Poverty is the level wherein an individual or people do not have the fundamental money-related assets and basics for the least expectation for everyday comforts. Therefore, this study applies a bayesian approach to poverty rates using the wealth index data in the south-western part of Nigeria to examine their poverty levels. The likelihood was Bernoulli and the conjugate Beta distribuitions at five different parameter values [Beta (1, 1), Beta (2, 2), Beta (4, 4), Beta (8, 8) and Beta (10, 10)] were elicited for the prior. Thus, the Beta-Bernoulli posteriors were derived, fitted and their parameters estimated for both the poor data set and the non-poor data set. The result for the poor data showed that as values of the prior parameters increases the posterior mean increases and the posterior variance decreases. So, at Beta (10, 10), the posterior standard variance is the lowest which indicates that about 36% of South-Western Nigeria population are extremely poor. Also, the result for the non poor data shows that as the values of the posterior parameters increases with increase in the prior parameters values, the posterior variance for prior, Beta (1, 1) has the least value 10.78%. This means that about 11% of South-Western Nigeria population are extremely non poor (rich).

Keywords: Poverty rate; beta-Bernoulli model; credible interval: prior distribution; posterior mean.

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1 Introduction

Citizens across the globe tend to classify themselves according to classes of society. Class is largely determined by the availability of financial power or wealth (poverty level). Poverty does not have enough material resources or income to meet the needs of an individual. Poverty may include elements of a societal, economic and political nature. The causes of poverty are a highly politically charged issue, because different causes point to different remedies. In general, the socialist tradition places the origins of poverty in the problems of distribution and the use of means of production as resources for the benefit of individuals, and calls for the redistribution of wealth as the solution. The main causes or primary causes of poverty are unemployment, inflation, poor utilization of resources, poor government policy or poor governance, debt, corruption, extreme weather, lack of access to education, overpopulation.

Since poverty can be said to be the level wherein an individual or people do not have the fundamental money-related assets and basics for the least expectation for everyday comforts, Poverty can be and or is measured by states, international organisations, policy-makers and professionals in various ways. Poverty is increasingly seen as multidimensional, encompassing social, cultural, and economic influences within larger socio-political systems. In 1990, the World Bank fixed the total poverty line as $1 a day. Ravallion [1] defined the poverty line as the minimum income level considered appropriate in a given country. International poverty line was updated by the World Bank to $1.90 a day as the global absolute minimum. By implication, anybody earning less than $1.90 a day is living in extreme poverty.

Table 1. Percentage of extremely poor people in the world by regions

Region	**$1 per day**			**$1.25 per day**	
	1990	**2002**	**2004**	**1981**	**2008**
East Asia and Pacific	15.4%	12.3%	9.1%	77.2%	14.3%
Europe and Central Asia	3.6%	1.3%	1.0%	1.9%	0.5%
Latin America and the Caribbean	9.6%	9.1%	8.6%	11.9%	6.5%
Middle East and North Africa	2.1%	1.7%	1.5%	9.6%	2.7%
South Asia	35.0%	33.4%	30.8%	61.1%	36%
Sub-Saharan Africa	46.1%	42.6%	41.1%	51.5%	47.5%
World				52.2%	22.4%

Deaton [2]

Dominique & Phong (2003): presents a Bayesian analysis of poverty rates in urban Ho Chi Minh City and rural Nghe An province in Vietnam. Their results rely on techniques due to Nandram and Sedransk [3] and Rahme et al [4], and make use of the software WINBUGS.

Zhou Xun and Michel Lubrano [5] provides a new estimation of an international poverty line based on a Bayesian approach. They found that the official poverty lines of the poorest countries are related to the countries' mean consumption level and proposed a new international poverty line at $1.48 per. Day (2005 PPP) worked on purchasing power parity (PPP) which was based on a reference group consumption level. Their figure was much higher than that proposed by the World Bank ($1.25 in 2005 PPP), but still within a reasonable confidence interval. By this standard, there are more than 1.7 billion people living in poverty.

Corey & Campbell [6]: made efforts to estimate various socio-demographic variables in small geographical areas which have proven difficult with the replacement of the Census long form with the American Community Survey (ACS). Kamaljit Kaur [7] worked on Bayesian estimators of Gini index and a Poverty measure obtained in case of Pareto distribution under censored and complete setup.

Idowu A. O. [8] concoluded that the poverty situation among the rural farm households in south western, Nigeria was found to be high (76.4 percent) and require 32.87 % of the poverty line (83.29/ day) to get out of poverty. An average rural farm household needed 253.39/person to meet the basic needs per day. He also discovered that Poverty was more severe among households whose heads were female, having low educational attainment and larger household size. Likewise, household size and dependency ratio entrenched the households' poverty while involvement in farm diversification and increase in educational level, size of land and investment assets owned by the household make the households to be less poor.

Olubusoye and Akanbi, [9] examined the correct specification of parameter priors which in the literature gives extremely low Posterior Model Probability (PMP), they decided to modify some existing parameter g-priors aimed at improving their sensitivities to PMP values and determining their predictive performances. It was later concluded that the modified g-priors performed better in model selection by their improved Posterior Model Probability values under the Bayesian Model Averaging framework. Also, they applied the best of the elicited priors to model the Nigerian Inflation process (data). Edwin & Lubrano [10] provided Bayesian inference for TIP curves, linking their expression to a parametric representation of the income distribution using a mixture of lognormal densities. They applied their methodology to evaluate the evolution of child poverty in Germany after 2002, providing thus an update the portrait of child poverty in Germany given in Corak et al. [11].

Olubusoye O. E. *et al* [12]: explores the spatial effect and the determinants of stunting in Nigeria. Using the 2013 Nigerian Demographic Health Survey (NDHS) data, they specified a model that simultaneously measures the fixed effect of categorical covariates, nonlinear effect of continuous variable, spatial effect and random effect of the community and households using the diffuse prior, the P-spline with second-order random walk, Markov random field prior and the exchangeable normal priors respectively. The dependent variable was specified as 1 if a child under five years (U5) is stunted and 0 otherwise. The logistic distribution was used to capture the binomial distribution of the dependent variable; the choices of hyperparameters were varied to check for the sensitivity of the priors on the posterior distribution.

Michel Lubrano *et al* [13]: used Bayesian to measure poverty in the developing world. They proposed a new methodology to revise the international poverty line (IPL) after (Ravallion et al. [14] using the same database, but augmented with new variables to take into account social inclusion in the definition of poverty along the lines of Atkinson and Bourguignon [15]. They worked on estimation of the world income distribution and of the corresponding number of poor people in the developing world. Their revised IPL is based on an augmented two- regime model estimated using a Bayesian approach, which allows them to take into account uncertainty when defining the reference group of countries where the IPL applies. The influence of weighting by population is discussed, as well as the IPL revision proposed in Deaton [2].

Dalal and Hall [16]: aprroximated priors by mixtures of natural conjugate priors. The theory of probability and history of statistics were extensively discussed in Laplace [17] and Stigler and Stephen [18] respectively. Causes of poverty were provided by Wikipedia [19]. William Bolstad [20] gave the insight of the Bayesian Statistics.

2 Methodology

The method adopted for this research was Bayesian approach: An application of the used of Bayes Theorem. The likelihood was Bernoulli (as the parameter of interest was poverty rate) while the elicited prior was Beta distribution.

2.1 Bayes theorem

$$P(\theta|y_i) = \frac{P(y_i|\theta)\times P(\theta)}{P(y_i)} \tag{1}$$

$P(y_i)$ is the normalizing constant of the posterior distribution. It is also the marginal distribution of y, and it is sometimes called the marginal distribution of the data.

2.2 Construction of likelihood

Since the dataset (y) for a trial is of two outcomes namely poor or non poor thus, it follows a bernoulli distribution:

$$y \sim \text{Bern}\,(\boldsymbol{\theta}),\ \ y=0, 1,\ \ 0< \boldsymbol{\theta} < 1 \tag{2}$$

Or

$$\boldsymbol{P(y|\theta)} = \boldsymbol{\theta^y(1-\theta)^{1-y}},\ \boldsymbol{0 < \theta <} 1 \tag{3}$$

Therefore, the likelihood is given as:

$$P(y_i|\theta) = \prod_{i=1}^{n} P(Y_i = y_i|\theta) \quad = \prod_{i=1}^{n} \theta^{y}(\mathbf{1-\boldsymbol{\theta}})^{1-y} \quad = \theta^{\sum y}(\mathbf{1-\boldsymbol{\theta}})^{n-\sum y} \tag{4}$$

2.3 Choosing a prior distribution

A natural conjugate prior for the Bernoulli likelihood is Beta distribution thus, Beta $(\boldsymbol{\alpha}, \boldsymbol{\beta})$.

In probability theory and statistics, the Beta distribution is a family of continuous probability distributions defined on the interval [0, 1] for the first kind and [0, ∞] for the second kind; Either is parameterized by two positive shape parameters, denoted by α and β, that appear as exponents of the random variable and control the shape of the distribution.

The Beta Distribution of the first kind with the random variable θ is defined as:

$$P(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \theta^{\alpha-1} (1-\theta)^{\beta-1} d\theta, \qquad 0< \theta < 1, \alpha > 0, \beta > 0 \tag{5}$$

The Beta Distribution of the second kind with the random variable θ is defined as:

$$P(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty \theta^{\alpha-1} (1+\theta)^{-(\alpha+\beta)} d\theta, \quad \theta > 0, \alpha > 0, \beta < \infty \tag{6}$$

$$\text{Where } \mathrm{B}(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \tag{7}$$

2.4 Posterior distribution for the beta prior

Combining Bernoulli likelihood in (4) and the Beta of the first kind in (5), the posterior distribution of the parameter θ given the dataset is given as:

$$p\left(\theta \middle| y_{i:\alpha,\beta}\right) = \frac{p(y_i|\theta) \times P(\theta)}{\int_0^1 p(y_i|\theta) \times P(\theta) d(\theta)} \tag{8}$$

Or

$$P(\theta|y_i) \propto P(y_i|\theta) \times P(\theta) \; = \; likelihood \times prior \tag{9}$$

Thus,

$$p\left(\theta \middle| y_{i:\alpha,\beta}\right) \propto \theta^{\sum y}(1-\theta)^{n-\sum y} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} \propto \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\sum y+\alpha-1}(1-\theta)^{n-\sum y+\beta-1}$$
$$\propto \frac{1}{\mathrm{B}(\alpha,\beta)} \theta^{\alpha+\sum y-1}(1-\theta)^{\beta+n-\sum y-1} \propto \; Beta\,(\alpha_n, \; \beta_n) \tag{10}$$

$where,$

$$\alpha_n = \alpha + \sum y \tag{11}$$

$$\beta_n = \beta + n - \sum y \tag{12}$$

2.5 Posterior mean for the beta prior

The Bayes estimate of the unknown parameter is simply the mean of the posterior distribution,

$$E(\theta|y_i) = \int_0^1 \theta \times p(\theta|y_i) d(\theta) \; = \; \int_0^1 \theta \times \frac{1}{\mathrm{B}(\alpha,\beta)} \; \theta^{\alpha+\sum y-1}(1-\theta)^{\beta+n-\sum y-1} d(\theta) \; = \; \frac{\alpha+\sum y}{\alpha+\beta+n} \tag{13}$$

2.6 Posterior variance for the parameter

The posterior variance for parameter is given as:

$$Var(\theta|y_i) = E(\theta^2|y_i) - (E(\theta|y_i))^2 \tag{14}$$

Then

$$E(\theta^2|y_i) = \int_0^1 \theta^2 \times p(\theta|y_i)d(\theta) \tag{15}$$

Therefore,

$$Var(\theta|y_i) = E(\theta^2|y_i) - (\frac{\alpha+\sum y}{\alpha+\beta+n})^2 \tag{16}$$

2.6.1 Credible interval for the posterior parameter

$$C.I(\theta|y) = E(\theta|y) \pm SE_{\alpha/2}(\theta|y) \tag{17}$$

3 Data Analysis and Discussion of Results

Secondary data obtained from wealth index which consist of classifications based on poor and non poor was used for the data analysis.

Table 2. Posterior mean and variance at different priors for the poor

Prior parameter	Post.mean	Post.var	C.I	
(1,1)	0.3333	0.0022	0.2456	0.4272
(2,2)	0.3365	0.0021	0.2494	0.4297
(4,4)	0.3426	0.0021	0.2565	0.4342
(8,8)	0.3534	0.0020	0.2694	0.4423
(10,10)	0.3583	0.0019	0.2753	0.4458

Table 2 shows that the posterior variance reduces with increasing prior hyper-parameters. At Beta (10,10), the posterior standard variance is the lowest in which about 35.83% of the population are extremely poor. Thus, between 27.53% - 44.58% of people are living in abject poverty in southwest Nigeria. The figures 1 – 5 below confirm the above claim.

Fitting Beta Bernoulli To The Data (Poor)

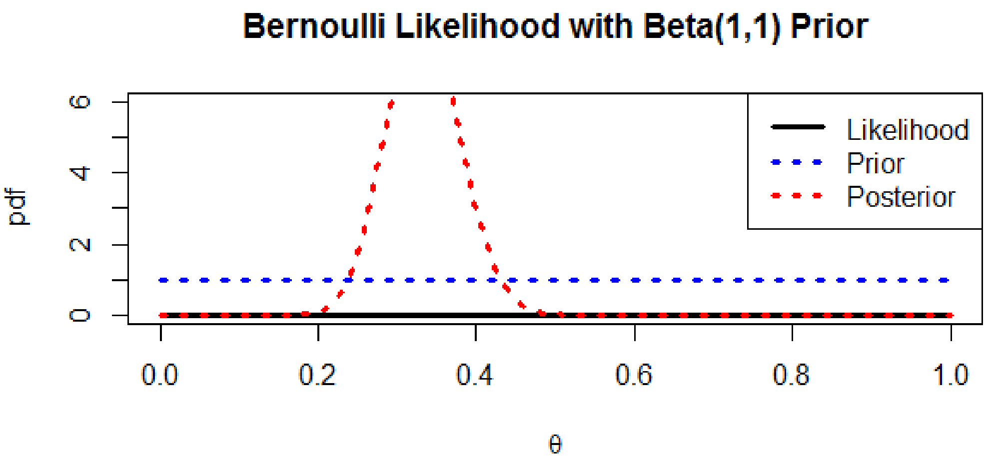

Fig. 1. Densities of the likelihood, beta (1, 1) and the posterior for the poor data

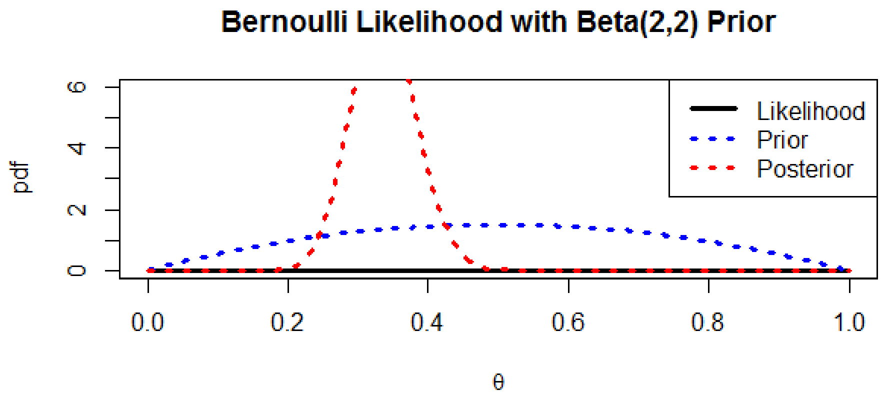

Fig. 2. Densities of the likelihood, beta (2, 2) and the posterior for the poor data

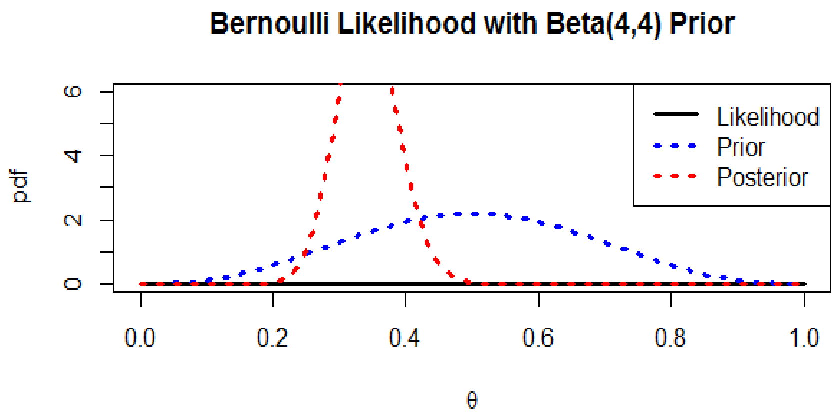

Fig. 3. Densities of the likelihood, beta (4, 4) and the posterior for the poor data

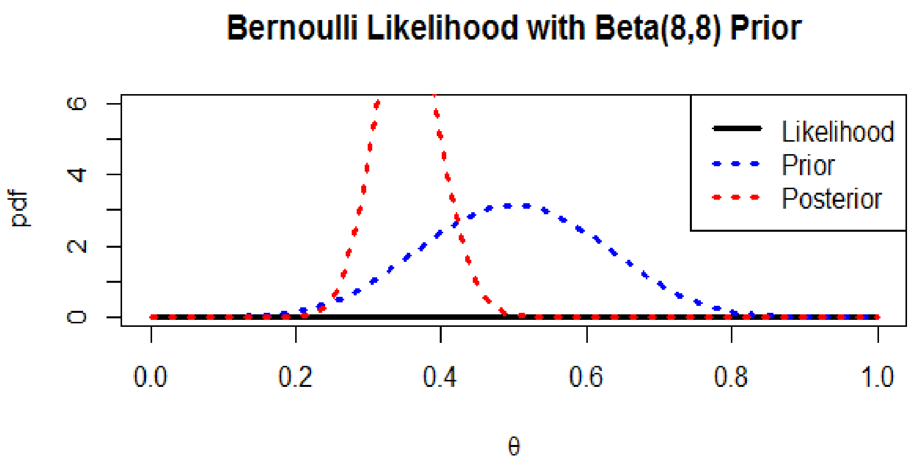

Fig. 4. Densities of the likelihood, beta (8, 8) and the posterior for the poor data

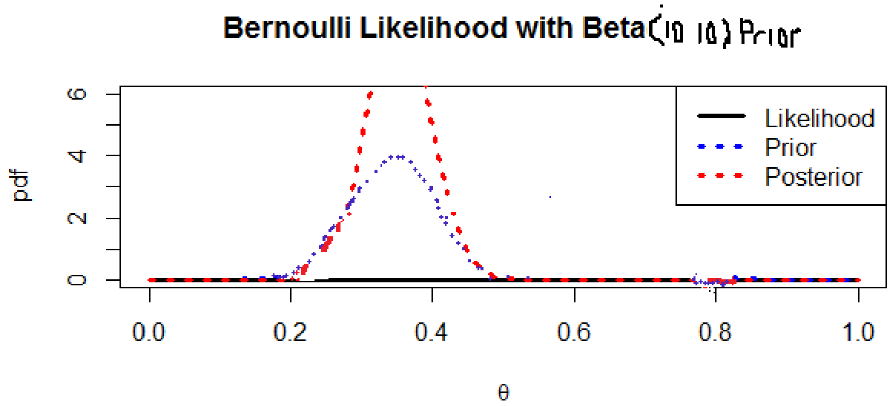

Fig. 5. Densities of the likelihood, beta (10, 10) and the posterior for the poor data

Table 3. Posterior mean and variance at different priors for the non poor

Prior Parameters	Post.mean	Post.var	C.I	
(1,1)	0.1078	0.0009	0.0556	0.1746
(2,2)	0.1154	0.0010	0.0617	0.1831
(4,4)	0.1296	0.0010	0.0734	0.1988
(8,8)	0.1552	0.0011	0.0955	0.2261
(10,10)	0.1667	0.0011	0.1058	0.2381

Table 3 shows that the posterior variance increases with increasing prior parameters. At Beta (1,1), the posterior standard variance is the lowest in which about 10.78% of the population are extremely rich. Similarly, between 5.56% - 17.46% are living in extreme wealth, influence and affluence in the south western part of Nigeria. Since the difference in percentage between non poor and poor people is so significant, it can be deduced that Nigeria is a poverty dwelling or developing country as less than 10% of the population controls the south-western country's wealth. Also, figures 6 – 9 depict the staement above.

3.1 Fitting beta – Bernoulli to the data (non poor)

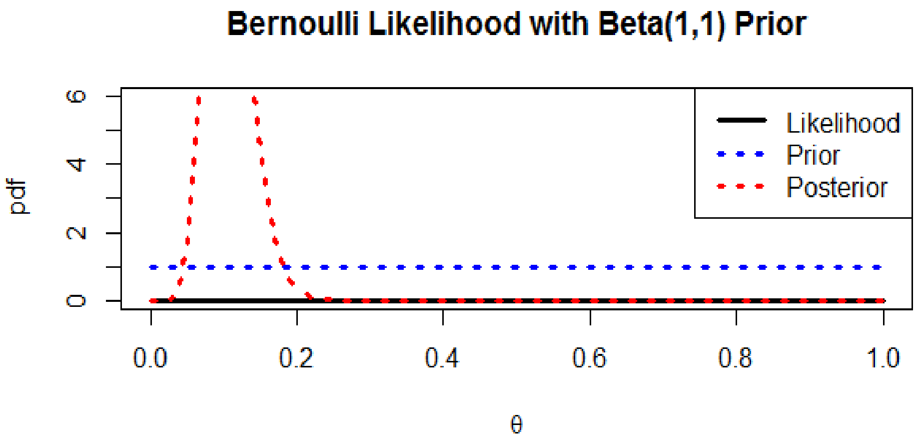

Fig. 6. Densities of the likelihood, beta (1, 1) and the posterior for the non poor data

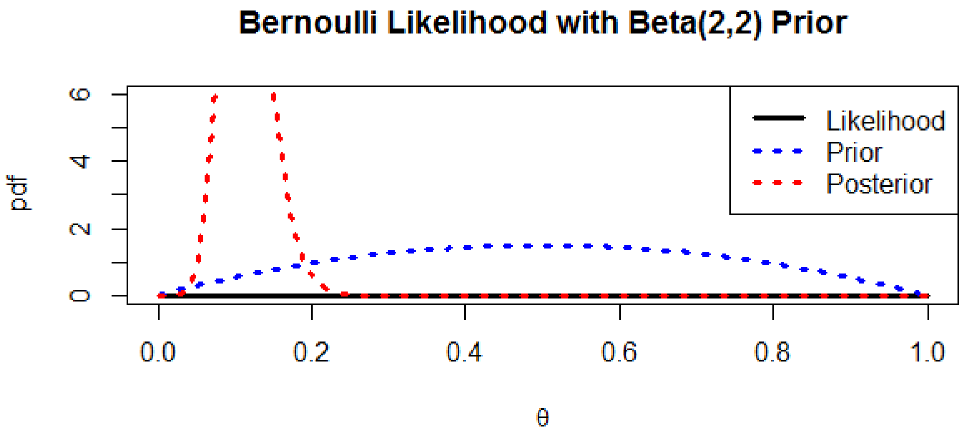

Fig. 7. Densities of the likelihood, beta (2, 2) and the posterior for the non poor data

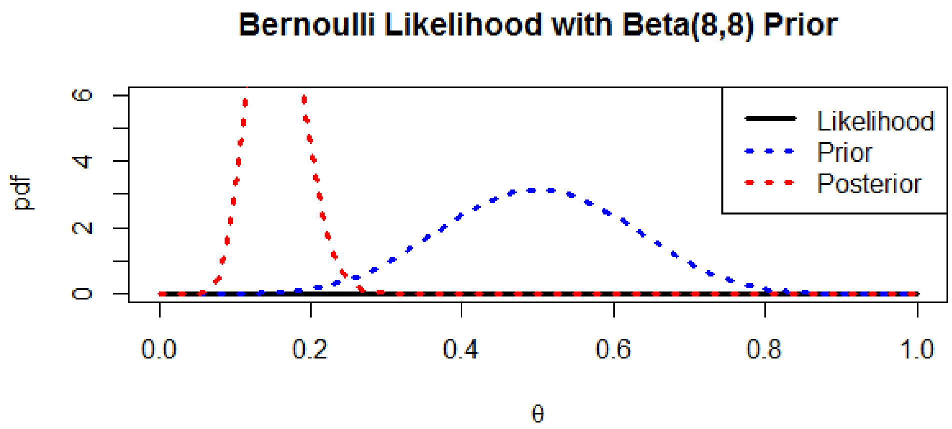

Fig. 8. Densities of the likelihood, beta (8, 8) and the posterior for the non poor data

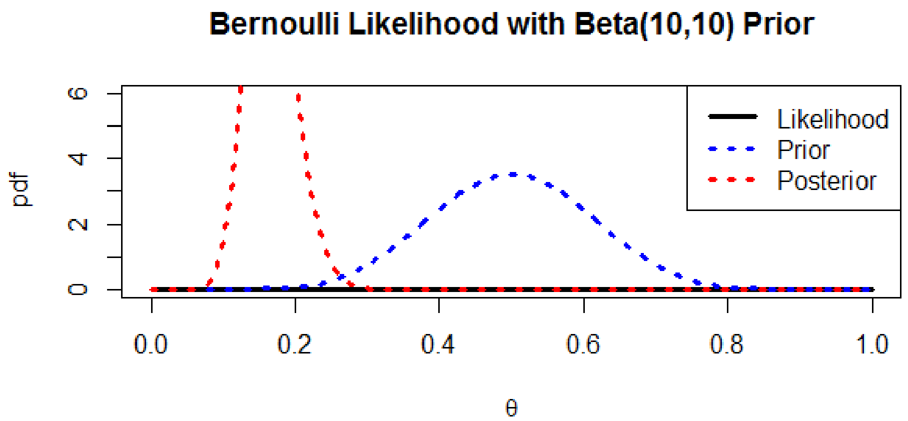

Fig. 9. Densities of the likelihood, beta (10, 10) and the posterior for the non poor data

4 Conclusion

The main objective of this research was to estimate poverty rate of South-Western part in Nigeria using Bayesian Approach. Bernoulli likelihood and Beta prior were used for the datasets to determine the poverty rate of South-Western part Nigeria. Beta-Bernoulli posterior was fitted and the parameters of the Beta-Bernoulli posterior were estimated for both the poor data set and the non poor data set at different values for the hyper-parameters of the Beta prior: Beta(1,1), Beta (2,2), Beta (4,4), Beta (8,8) and Beta(10,10)).

The results for the poor data showed that as values of the prior parameters increases the mean and the variance of the posteriors increases and decreases respectively. So at Beta (10, 10), the posterior standard variance is the lowest with the posterior mean of 35.83% which indicates that about 36% of people leaving in the South-Western part of Nigeria are extremely poor. Also the results for the non poor data show that as the values of the parameters of the prior increases the values of the posterior parameters (i.e. Mean and Variance) increases. The posterior standard variance of prior at Beta (1, 1) is the least with the posterior mean of 10.78%. This means that about 11% of people leaving in the South-Western part of Nigeria are extremely rich.

The results further showed that between 27.53% and 44.58% are people leaving in the South-Western part of Nigeria with abject poverty while just between 5.56% to 17.46% are living in extreme wealth, influence and affluence. Thus, the government should develop and implement rapid and sustained economic growth policies and programs, in south western part of the country, such as health, education, nutrition and sanitation, allowing the poor to participate and contribute to the growth. Studies show that a 10% increase in a country's average income reduces poverty by as much as 20-30%. Similarly, the Government should also initiate poverty programmes, which should be all-inclusive and properly monitored to ensure that such programmes reached the desired or targeted population in South-Western part of Nigeria. Finally, this research is useful to the academicians, policy makers and development practitioners.

Competing Interests

Author has declared that no competing interests exist.

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