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Generalized Inverse Power Sujatha Distribution with Applications

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, we present a new lifetime distribution known as the generalized inverse power Sujatha distribution. The statistical and mathematical properties of the new distribution such as the moment and moment generating function, Renyi entropy and distribution of order statistics have been derived and discussed. Also, reliability measures like survival function, hazard function, reverse hazard rate, cumulative hazard rate and odds function are discussed. Maximum likelihood estimation technique was used to estimate the parameters. However, a 95% confidence intervals were constructed for the parameters. Finally, we applied the proposed distribution to two lifetime datasets and compare its superiority over other candidate models. Results obtained indicates that the generalized inverse power Sujatha distribution outperform the other competing models.

Keywords: *Sujatha distribution; inverse Sujatha distribution; generalized inverse power sujatha distribution; maximum likelihood estimation.*

Mathematics Subject Classification: 60E05. 62E15. 65C05.

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1 Introduction

In anattempt to model life data sets, many lifetime distributions have been proposed. It is often believed that lifetime data sets assume a specific parametric statistical distribution. In most appliedconditions, most of the famous statistical distributions do not provide a good fit to data set; hence, one is steered to search for some adjustments of existing distributions that can model aptly, the data sets. Recent studies have shown that a lot of statistical distributions have been developed by researchers using different methods of generating distributions.These families of distributions are gotten by introducing one or more additional shape parameter(s) to the baseline distribution.One can see some of the methods in the following articles: the beta-G family by Eugene et al. [1], transmuted family of distribution by Shaw and Buckley [2], gamma-G (type 1) family by Zografos and Balakrishnan [3], Kumaraswamy-G family by Cordeiro, and De Castro [4], transformed-transformer (T-X) by Alzaatreh et al. [5], Weibull-generated (KwW-G), type II half logistic–G family introduced by Hassan et al. [6], gamma-G family, type 2, by Risti ́c and Balakrishnan [7],The Zubair-G Family of Distributions by Zubair [8], and others.

Mudholkar and Srivastava [9] introduced the exponentiation method. The approach was employed to define the exponentiatedWeibull distribution. Suppose X is a random variable with a baseline cumulative density function, cdf $F(x)$. Mudholkar and Srivastava defined the new family of distributions with the form

$$F(x,\beta) = [G(x,\beta)]^{\beta};\ x \in R\ ;\ \beta > 0 \tag{1}$$

where for $\beta = 1$, (1) reduces to the cdf of the baseline distribution.

The corresponding probability density function, pdf was obtained by taken the first derivative of (1). Consequently, we have

$$f(x,\beta) = \beta[G(x,\beta)]^{\beta-1} f(x,\beta);\ x \in R\ ;\ \beta > 0 \tag{2}$$

A completehandling of the general properties of the family defined by (1) was carried out by Gupta et al. [10]. This method of generating new distribution has been used by many researchers. Notable among them are the exponentiated exponential distribution by Zakaria and AL-Jammal [11] used to study the failure time data from two machines. Rameesa [12] Proposed exponentiated inverse power Lindley distribution and used it to model the active repair times (hours) for an airborne communication transceiver. Gupta and Kundu [13] developed the generalized exponential distribution. The distribution has a right skewed unimodal density function and monotone hazard function, and can be used to model life time data in place of Weibull, Log-normal and gamma distribution. For other articles on exponentiation, see Nwankwo et al. [14], Onyekwere et al. [15], Elgarhy and Shawki [16], Okereke and Uwaeme [17], among others. The purpose of this research is to provide a generalized inverse power Sujatha distribution capable of modelling data sets with non-monotone hazard rates.

1.1 Sujatha distribution

Shanker [18] proposed a new continuous distribution, which is better than Akash, Shanker, Lindley and exponential distributions for modelling lifetime data by considering a three-component mixture of an exponential distribution with scale parameter θ, a gamma distribution with shape parameter 2 and scale parameter θ, and a gamma distribution with shape parameter 3, and scale parameter θ with their mixing proportions $\frac{\theta^2}{\theta^2+\theta+2}$, $\frac{\theta}{\theta^2+\theta+2}$ and $\frac{2}{\theta^2+\theta+2}$ respectively. The probability density function (p.d.f) of the new one-parameter lifetime distribution can be introduced as

$$f(y,\theta) = \frac{\theta^3}{\theta^2+\theta+2}(1+y+y^2)e^{-\theta y}\ _{;\, y>0,\ \theta>0} \tag{3}$$

The corresponding cumulative distribution function (c.d.f) of Sujatha distribution was obtained as

$$F(y,\theta) = 1 - \left[1 + \frac{\theta y(\theta y+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta y}\ _{;\, y>0, \theta>0} \tag{4}$$

The kth moment of the Sujatha distribution is defined by

$$\mu_r' = \frac{r!\left[\theta^2+(r+1)\theta+(r+1)(r+2)\right]}{\theta^r(\theta^2+\theta+2)}; r = 1,2,3,4 \quad (5)$$

The first and second crude moments are respectively given by

$$\mu_1' = \frac{\theta^2+2\theta+6}{\theta(\theta^2+\theta+2)} \text{ and } \mu_2' = \frac{2(\theta^2+3\theta+12)}{\theta^2(\theta^2+\theta+2)}$$

The lifetime distribution proposed is unimodal and capable of modelling lifetime data with increasing function in their hazard rate. The hazard rate function of Sujatha distribution is defined by

$$h(x) = \frac{\theta^2(1+x+x^2)}{\theta^2(1+x+x^2)+2\theta x+\theta+2} \quad (6)$$

The rest of the paper is organized as follows: section 2 introduces the Inverse Power Sujatha distribution, section 3 introduce also the Generalized Inverse Power Sujatha distribution, section 3.1 presents the mathematical and statistical characteristics of generalized Inverse Power Sujatha distribution, section 3.2 presents the maximum likelihood estimation, section 4 contains the numerical applications, section 5, are results and discussion. In section 6, we conclude the article.

2 Inverse Power Sujathadistribution

Definition 1: Let X be a random variable from a continuous distribution such that, $X \sim IPS(x, \alpha, \theta)$, then the pdf and cdf are respectively given by

$$f_{IPS}(x,\alpha,\theta) = \frac{\alpha\theta^3}{\theta^2+\theta+2}(1 + x^{-\alpha} + x^{-2\alpha})x^{-(\alpha+1)}e^{-\theta x^{-\alpha}}; x > 0, \alpha, \theta > 0 \quad (7)$$

$$F_{IPS}(x,\alpha,\theta) = \left[1 + \frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}} \quad ; x > 0, \alpha, \theta > 0 \quad (8)$$

Corollary 1: The Inverted Power Sujatha Distribution is a valid probability density function. Thus,

$$\begin{cases} \int_0^\infty f_{IPS}(x,\alpha,\theta)\,dx = 1 \quad \text{or} \\ \lim_{x\to\infty} F_{IPS}(x,\alpha,\theta) = 1 \end{cases}$$

To show that $f_{IPS}(x, \alpha, \theta)$ is a valid pdf, we recall that

$$\int_0^\infty f(x,\alpha,\theta)\,dx = 1 \quad or \lim_{x\to 0} F(x,\alpha,\theta) = 1$$

Thus,

$$\frac{\alpha\theta^3}{\theta^2+\theta+2}\int_0^\infty (1 + x^{-\alpha} + x^{-2\alpha})x^{-(\alpha+1)}e^{-\theta x^{-\alpha}}\,dx = 1 \quad (9)$$

Proof:

By letting= $x^{-\alpha}$, applying the right transformation, and substituting in (9), we get

$$= \frac{\theta^3}{\theta^2+\theta+2}\int_0^\infty (1 + y + y^2)e^{-\theta y}\,dy$$
$$= \frac{\theta^3}{\theta^2+\theta+2}\left[\int_0^\infty e^{-\theta y}\,dy + \int_0^\infty ye^{-\theta y}dy + \int_0^\infty y^2e^{-\theta y}\,dy\right]$$
$$= \frac{\theta^3}{\theta^2+\theta+2}\left[\frac{\theta^2+\theta+2}{\theta^3}\right] = 1$$

Thus, $f_{IPS}(x, \alpha, \theta)$ is a valid probability density function (pdf)

3 Generalized Inverse Power Sujatha Distribution

Definition 2: A random variable $X$with pdf and cdf defined in (7) and (8), is said to have generalized inverse power Sujatha distribution if thecdf and pdf are respectively given by:

Thus;

$$F_{GIPS}(x,\alpha,\theta,\beta) = \left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta} \tag{10}$$

$$f_{GIPS}(x,\alpha,\theta,\beta) = \left\{ \begin{matrix} \frac{\beta\alpha\theta^3}{\theta^2+\theta+2}(1+x^{-\alpha}+x^{-2\alpha})x^{-(\alpha+1)} \\ \times\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta-1} e^{-\theta x^{-\alpha}} \end{matrix} \right\}, x>0;\ \alpha,\beta,\theta \tag{11}$$

Equations (10) and (11) are obtained by substituting (7) into (2) and (8) into (1).

Fig. (1a), (1b), (1c) and (1d) below show the graphs of the probability density function and cumulative density function of Generalized Inverse Power Sujatha distribution for some values of the parameter θ. The graph was plotted using R software version 4.0.3.

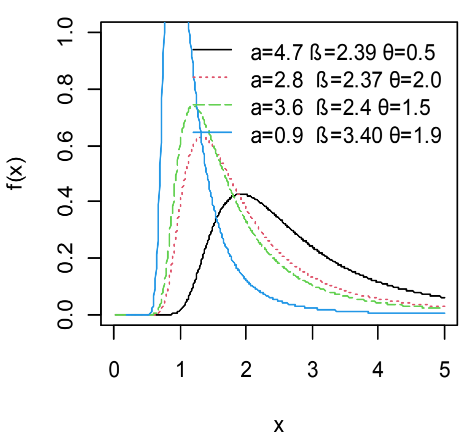


Fig. 1a. pdf plot of GIPS

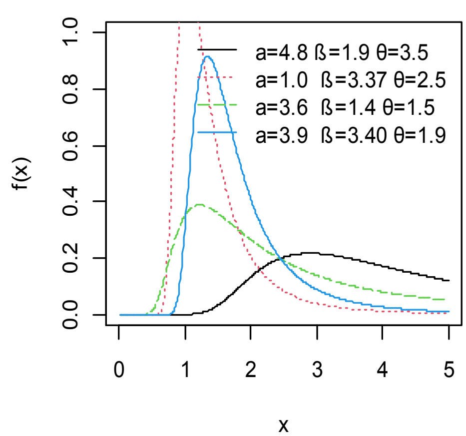
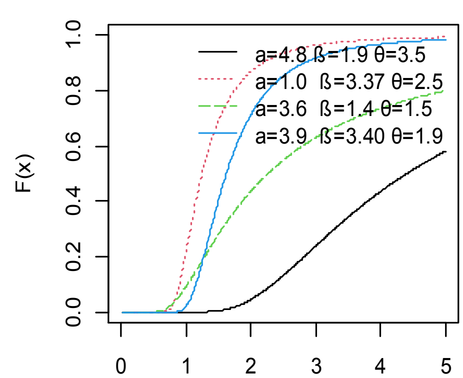


Fig. 1b. pdf plot of GIPS



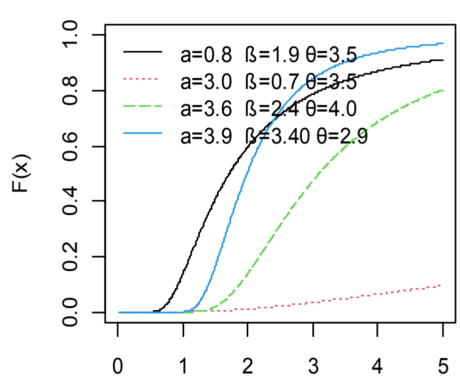


Fig. 1c.cdf of GIPS Fig. 1d.cdf of GIPS

From the pdf of generalized inverse power Sujatha distribution given in (11), we can deduce the following

1. When $\beta = 1$, the distribution reduced to Inverse Power Sujatha distribution.
   $f(x,\alpha,\theta) = \frac{\alpha\theta^3}{\theta^2+\theta+2}(1+x^{-\alpha}+x^{-2\alpha})x^{-(\alpha+1)}e^{-\theta x^{-\alpha}}$
2. If $\alpha = 1$ and $\beta = 1$, the distribution reduces to Inverse Sujatha distribution
   $f(x,\theta) = \frac{\theta^3}{\theta^2+\theta+2}\left(\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}\right)e^{-\frac{\theta}{x}}$; $x > 0, \theta > 0$
3. If $\alpha = 1, \beta = 1$ and $y = x^{-1}$, the GIPS distribution returns to the baseline distribution given in equation (3).

3.1 Mathematical characteristics of generalized Inverse power sujatha distribution

3.1.1 Moments

Definition 3: Given a random variable X from a continuous univariate distribution, the rth moment is given by:

$$E(X^r) = \int_0^\infty x^r f(x)\,dx \tag{12}$$

Inserting (11) into (12), one obtains

$$= \left\{ \frac{\alpha\beta\theta^3}{\theta^2+\theta+2} \int_0^\infty x^r \left(1+x^{-\alpha}+x^{-2\alpha}\right) x^{-(\alpha+1)} \times \left(\left(1+\frac{\theta x^{-\alpha}\left(\theta x^{-\alpha}+\theta+2\right)}{\theta^2+\theta+2} \right) e^{-\theta x^{-\alpha}} \right)^{\beta-1} e^{-\theta x^{-\alpha}}\,dx \right\} \tag{13}$$

where $\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta-1}$

$$= \sum_{i=0}^{\infty}\binom{\beta-1}{i}\frac{\theta^i x^{-i\alpha}}{(\theta^2+\theta+2)^i}\sum_{j=0}^{i}\binom{i}{j}\theta^{i-j}x^{-\alpha i+\alpha j}\sum_{k=0}^{j}\binom{i}{k}\theta^k 2^{j-k}e^{-i\theta x^{-\alpha}} \tag{14}$$

Substituting equation (14) into equation (13), and simplifying, we get

$$= \left\{ \begin{array}{l} \sum_{i=0}^{\infty}\binom{\beta-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{i}{k}\frac{2^{j-k}\alpha\beta\theta^{2i-j+k+3}}{(\theta^2+\theta+2)^{i+1}}\int_0^\infty x^{r+\alpha j-2\alpha i-\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \\ +\sum_{i=0}^{\infty}\binom{\beta-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{i}{k}\frac{2^{j-k}\alpha\beta\theta^{2i-j+k+3}}{(\theta^2+\theta+2)^{i+1}}\int_0^\infty x^{\alpha j+r-2\alpha i-2\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \\ +\sum_{i=0}^{\infty}\binom{\beta-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{i}{k}\frac{2^{j-k}\alpha\beta\theta^{2i-j+k+3}}{(\theta^2+\theta+2)^{i+1}}\int_0^\infty x^{\alpha j+r-2\alpha i-3\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \end{array} \right\} \tag{15}$$

Let $\Lambda_{i,j,k} = \sum_{i=0}^{\infty}\binom{\beta-1}{i}\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{j}\binom{i}{k}\frac{2^{j-k}\alpha\beta\theta^{2i-j+k+3}}{(\theta^2+\theta+2)^{i+1}}$

Consequently, we have

$$= \Lambda_{i,j,k}\,\alpha \left\{ \begin{array}{l} \int_0^\infty x^{r+\alpha j-2\alpha i-\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \\ +\int_0^\infty x^{r+\alpha j-2\alpha i-2\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \\ +\int_0^\infty x^{r+\alpha j-2\alpha i-3\alpha-1}e^{-\theta(i+1)x^{-\alpha}}\,dx \end{array} \right\} \tag{16}$$

If $y = x^{-\alpha}$, by applying transformation techniques and substituting in equation (16), we get

$$= \Lambda_{i,j,k} \left\{ \begin{array}{l} \int_0^\infty y^{\frac{r}{\alpha}+j-2i-2}\, e^{-\theta(i+1)y}\, dy \\ + \int_0^\infty y^{\frac{r}{\alpha}+j-2i-3}\, e^{-\theta(i+1)y}\, dy \\ + \int_0^\infty y^{\frac{r}{\alpha}+j-2i-4}\, e^{-\theta(i+1)y}\, dy \end{array} \right\} \quad (17)$$

Applying the gamma transformation $\int_0^\infty \frac{e^{-\beta/t}}{x^{\alpha+1}} dx = \frac{\Gamma(\alpha)}{\beta^\alpha}$, to equation (17) and simplifying, we get the rth moment of GIPS. Thus

$$E(X^r) = \Lambda_{i,j,k} \left[\frac{\Gamma\left(\frac{r}{\alpha}+j-2i-1\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-1\right)}} + \frac{\Gamma\left(\frac{r}{\alpha}+j-2i-2\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-2\right)}} + \frac{\Gamma\left(\frac{r}{\alpha}+j-2i-3\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-3\right)}} \right] \quad (18)$$

3.1.2 Moment generating function of generalized inverse power Sujatha distribution

Definition 4: Given a random variable X, from a continuous univariate distribution, the moment generating function, $M_X(t)$ is

$$M_X(t) = E[e^{tX}] = \int_0^\infty e^{tx} f(x)\, dx \quad (19)$$

$$= \int_0^\infty \sum_{n=0}^\infty \frac{(tx)^n}{n!} f(x)\, dx$$

$$= \sum_{n=0}^\infty \frac{t^n}{n!} \int_0^\infty x^n\ f(x)\, dx$$

$$= \sum_{n=0}^\infty \frac{t^n}{n!} E(x^n) \quad (20)$$

Where $E(x^n) = E(x^r)$, the rth moment of a distribution. Thus, substituting for $E(x^r)$ in eqn (20), we obtain

$$M_X(t) = \sum_{n=0}^\infty \frac{t^n}{n!} \left[\Lambda_{i,j,k} \left[\frac{\Gamma\left(\frac{r}{\alpha}+j-2i-1\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-1\right)}} + \frac{\Gamma\left(\frac{r}{\alpha}+j-2i-2\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-2\right)}} + \frac{\Gamma\left(\frac{r}{\alpha}+j-2i-3\right)}{[\theta(i+1)]^{\left(\frac{r}{\alpha}+j-2i-3\right)}} \right] \right] \quad (21)$$

3.1.3 Survival function generalized inverse power Sujatha distribution

The survival function denoted by $R(x)$ describe the probability that an individual survives beyond time x. It is a significant instrument used in reliability investigations. It can be estimated using the nonparametric Kaplan-Meier curve or one of the parametric distribution functions.

Let X be a positive random variable from a distribution with probability density function $f(x)$ and cumulative distribution function $F(x)$. Thus, the reliability or survival function as defined in Zhou [20] is

$$S_{GIPS}(x,\alpha,\theta,\beta) = p(X > x) = 1 - p(X \le x) = 1 - F_{GIPS}(x,\alpha,\theta,\beta) \quad (22)$$

where $F(x) = p(X \le x) = \int_{-\infty}^{x} f(x)dx$

Survival function $R(x)$ is monotone decreasing over the interval $[0,\infty)$, $\lim_{x\to 0} R(x)=1$, implies a proper functioning system, while $\lim_{x\to\infty} R(\infty)=0$, means that the no system remains working forever. Substituting (10) in (22), we get

$$S_{GIPS}(x,\alpha,\theta,\beta) = 1 - \left[\left[1 + \frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right] e^{-\theta x^{-\alpha}}\right]^{\beta} \tag{23}$$

3.1.4 Hazard function generalized inverse power sujatha distribution

Given a random variable X from a continuous distribution, the hazard rate $h(x)$ is given by

$$h(x) = \lim_{\Delta x\to 0} \frac{F(x+\Delta x)-F(x)}{\Delta x R(x)} = \frac{f(x)}{R(x)} \qquad \text{Zhou [21]} \tag{24}$$

Hence, the hazard rate of the generalized inverse power Sujatha distribution is

$$h(x,\alpha,\theta,\beta) = \frac{\frac{\beta\alpha\theta^3}{\theta^2+\theta+2}(1+x^{-\alpha}+x^{-2\alpha})x^{-(\alpha+1)}\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta-1} e^{-\theta x^{-\alpha}}}{1-\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta}} \tag{25}$$

The behaviour of the survival and the hazard functions of GIPS distribution for varying values of parameter $\beta, \alpha, \text{and}\, \theta$ are shown in Fig. 2a, 2b, 2c and 2d, respectively. The graph was plot using R software version 4.0.3.

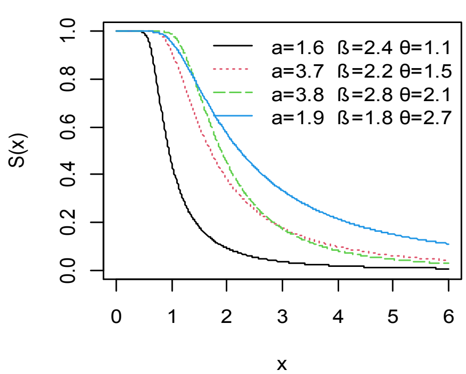


Fig. 2a. Survival plot of GIPS

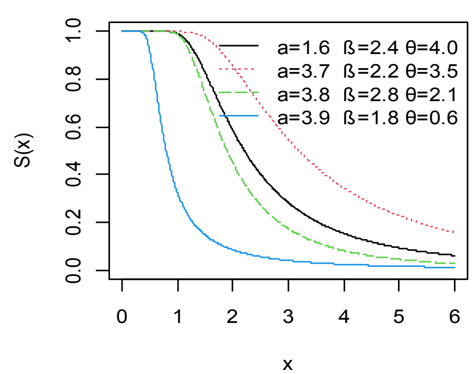


Fig. 2b. Survival plot of GIPS

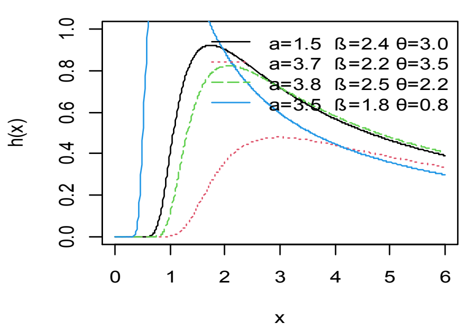


Fig. 2c. Hazard function of GIPS

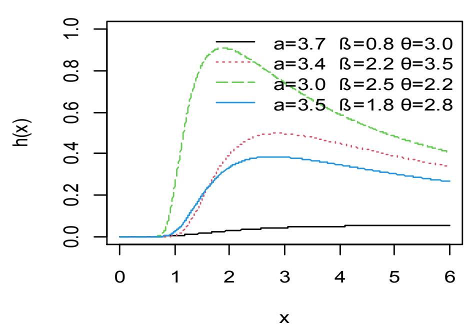


Fig. 2c. Hazard function of GIPS

3.1.5 Cumulative hazard function

A random variable X from a univariate continuous distribution is said to have cumulative hazard function if $H(x,\alpha,\theta,\beta)$ is defined by:

$$H(x,\alpha,\theta,\beta) = -ln[1-F(x)] \tag{26}$$

Inserting (10) in (26), we get the following

$$H(x,\alpha,\theta,\beta) = -ln\left[1-\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta} \tag{27}$$

In relation to probability plot, the cumulative hazard plots help to visually examine the distributional model assumptions for reliability data. It has the same interpretation as the probability plot.

3.1.6 Reverse hazard rate

The reversed hazard rate $\Omega_\tau(x)$ of a system is describingthe instantaneous conditional probability that the system has survived the instant $(x-\Delta x)$, given that the system has failed before time x. Reversed hazard rate is given by

$$\Omega_\tau(x) = \frac{f(x)}{F(x)} \tag{28}$$

Using equation (28) and substituting for $f(x)$ and $F(x)$, we obtain the reverse hazard function as

$$\Omega_\tau r(x) = \frac{\frac{\beta\alpha\theta^3}{\theta^2+\theta+2}(1+x^{-\alpha}+x^{-2\alpha})x^{-(\alpha+1)}\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta-1}e^{-\theta x^{-\alpha}}}{\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta}} \tag{29}$$

3.1.7 Odds function

The odds function of generalise inverse power Sujatha distribution is given by

$$O(x) = \frac{F(x,\alpha,\theta,\beta)}{1-F(x,\alpha,\theta,\beta)} \tag{30}$$

$$O(x) = \frac{\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta}}{1-\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta}} \tag{31}$$

3.1.8 Renyi entropy of GIPS distribution

Entropy is very beneficial in determining the uncertainty of a probability distribution**.** Entropy has uses in economics, probability and statistics, communication theory and so on. Large value of entropy signifies large uncertainty in the data. Re' nyi [22] gave an expression for any given probability distribution. The R$\acute{e}$nyi entropy of a random variable X from a continuous distribution is given by

Definition 5: Suppose X is a random variable that follows GIPS distribution with cdf and pdf defined in (10) and (11), the Renyi entropy is given by

$$T_\varphi = \frac{1}{1-\varphi}\log\left\{\int_R f_{(x)}^{\varphi}\,dx \qquad where\ \varphi > 0\ and\ \varphi \neq 1 \tag{32}$$

Therefore,

$$T_\varphi(x,\alpha,\beta,\theta)=\frac{1}{1-\varphi}log\left\{\int_0^\infty\left[\begin{array}{c}\frac{\beta\alpha\theta^3}{\theta^2+\theta+2}(1+x^{-\alpha}+x^{-2\alpha})x^{-(\alpha+1)}\\ \times\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\beta-1}e^{-\theta x^{-\alpha}}\end{array}\right]^\varphi dx\right\} \tag{33}$$

$$=\frac{1}{1-\varphi}\log\left\{\int_0^\infty\left[\begin{array}{c}\frac{\beta^\varphi\alpha^\varphi\theta^{3\varphi}}{(\theta^2+\theta+2)^\varphi}\sum_{i=0}^\infty\binom{\varphi}{i}x^{\varphi-i}\sum_{j=0}^\infty\binom{i}{j}x^{-j\alpha}x^{-\varphi(\alpha+1)}\\ \times\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\varphi(\beta-1)}e^{-\varphi\theta x^{-\alpha}}\end{array}\right]dx\right\} \tag{34}$$

Where,

$$\left[\left[1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x^{-\alpha}}\right]^{\varphi(\beta-1)}$$

$$=\left\{\begin{array}{c}\sum_{k=0}^\infty\binom{\varphi(\beta-1)}{k}\frac{\theta^k x^{-K\alpha}}{(\theta^2+\theta+2)^k}\sum_{l=0}^k\binom{k}{l}(\theta x^{-\alpha})^{k-l}\\ \times(\theta+2)^l\sum_{m=0}^l\binom{l}{m}\theta^{l-m}2^m e^{-k\theta x^{-\alpha}}\end{array}\right\} \tag{35}$$

Substituting (35) into (34), we have

$$=\frac{1}{1-\varphi}log\left\{\begin{array}{c}\sum_{j=0}^\infty\binom{\varphi}{i}\sum_{j=0}^\infty\binom{i}{j}\sum_{k=0}^\infty\binom{\varphi(\beta-1)}{k}\sum_{l=0}^k\binom{k}{l}\\ \times\sum_{m=0}^l\binom{l}{m}\frac{2^m\alpha^\varphi\beta^\varphi\theta^{3\varphi+2k-m}}{(\theta^2+\theta+2)^{\varphi+k}}\int_0^\infty x^{\alpha l-i-\alpha j-\alpha\varphi-2\alpha k}e^{-\theta(k+\varphi)x^{-\alpha}}dx\end{array}\right\} \tag{36}$$

Let $=x^{-\alpha}$, appropriate transformation and substitution gives

$$=\frac{1}{1-\varphi}log\left\{\begin{array}{c}\sum_{j=0}^\infty\binom{\varphi}{i}\sum_{j=0}^\infty\binom{i}{j}\sum_{k=0}^\infty\binom{\varphi(\beta-1)}{k}\\ \times\sum_{l=0}^k\binom{k}{l}\sum_{m=0}^l\binom{l}{m}\frac{2^m\alpha^{\varphi-1}\beta^\varphi\theta^{3\varphi+2k-m}}{(\theta^2+\theta+2)^{\varphi+k}}\\ \times\int_0^\infty y^{\frac{i}{\alpha}-\frac{1}{\alpha}-j+2k-l+\varphi+1}e^{-\theta(k+\varphi)y}dy\end{array}\right\} \tag{37}$$

Let $A_{i,j,k,l,m}=\left\{\begin{array}{c}\sum_{j=0}^\infty\binom{\varphi}{i}\sum_{j=0}^\infty\binom{i}{j}\sum_{k=0}^\infty\binom{\varphi(\beta-1)}{k}\\ \times\sum_{l=0}^k\binom{k}{l}\sum_{m=0}^l\binom{l}{m}\frac{2^m\alpha^{\varphi-1}\beta^\varphi\theta^{3\varphi+2k-m}}{(\theta^2+\theta+2)^{\varphi+k}}\end{array}\right\}$

Substituting for $A_{i,j,k,l,m}$ in equation (37), we have the Renyi entropy of GIPS distribution.

$$T_\varphi(x,\alpha,\beta,\theta)=\frac{1}{1-\varphi}log\left[A_{i,j,k,l,m}\frac{\Gamma\left(\frac{i}{\alpha}+\frac{1}{\alpha}+j+2k-l+\varphi+2\right)}{[\theta(k+\varphi)]^{\left(\frac{i}{\alpha}+\frac{1}{\alpha}+j+2k-l+\varphi+2\right)}}\right] \tag{38}$$

3.1.9 Distribution of Order Statistics for GIPS distribution

Consider a random sample $X_1, X_2, X_3 \dots X_n$ from the GIPS distribution with pdf $f(x)$ and cdf $F(x)$. Let $x_{(1)}, x_{(2)}, x_{(3)} \dots x_{(n)}$ be the corresponding order statistics. Thus, the pdf of the kth order statistics defined by Hogg and Craig [23] is given by

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!} f(x)\big(F(x)\big)^{k-1}\big(1-F(x)\big)^{n-k} \tag{39}$$

With binomial expansion, the simplification of (3.50) gives

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!} \sum_{\omega=0}^{\Pi-k} \binom{n-k}{\omega} (-1)^w F^{k+w-1}(x) f(x) \tag{40}$$

Substituting for $f(x)$ and $F(x)$ of GIPS distribution, one obtains

$$f_{X_{(k)}}(x) = \frac{n!\beta\alpha\theta^3(1+x^{-\alpha}+x^{-2\alpha})}{(\theta^2+\theta+2)(k-1)!(n-k)!} x^{-(\alpha+1)} e^{-\theta x^{-\alpha}}$$
$$\times \sum_{\omega=0}^{\Pi-k} \binom{n-k}{\omega} (-1)^w \left(\left(1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right) e^{-\theta x^{-\alpha}}\right)^{\beta(k+\omega)-1} \tag{41}$$

For $k = 1$, we have the pdf of the first order statistics $X_{(1)}$ defined by

$$f_{X_{(1)}}(x) = \begin{Bmatrix} \frac{n!\beta\alpha\theta^3(1+x^{-\alpha}+x^{-2\alpha})}{(\theta^2+\theta+2)(n-1)!} x^{-(\alpha+1)} e^{-\theta x^{-\alpha}} \\ \times \sum_{\omega=0}^{\Pi-k} \binom{n-1}{\omega} (-1)^\omega \left(\left(1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right) e^{-\theta x^{-\alpha}}\right)^{\beta(1+\omega)-1} \end{Bmatrix} \tag{32}$$

Also, for $k = n$, we have the pdf of the nth order statistics $X_{(n)}$ as

$$f_{X_{(n)}}(x) = \begin{Bmatrix} \frac{n!\beta\alpha\theta^3(1+x^{-\alpha}+x^{-2\alpha})}{(\theta^2+\theta+2)(n-1)!} x^{-(\alpha+1)} e^{-\theta x^{-\alpha}} \\ \times \left(\left(1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right) e^{-\theta x^{-\alpha}}\right)^{\beta(n+\omega)-1} \end{Bmatrix} \tag{33}$$

The cumulative density function the kth order statistics is defined by

$$F_{X_{(k)}}(x) = \sum_{i=k}^{n} \binom{n}{i} F^i(x)\big(1-F(x)\big)^{n-i} \tag{34}$$

Expanding (34) further using binomial expansion, we have

$$F_{X_{(k)}}(x) = \sum_{\mathrm{i=k}}^{\mathrm{n}} \binom{n}{i} \sum_{j=0}^{n-j} \binom{n-i}{j} (-1)^j F^{i+j}(x) \tag{35}$$

Substituting for $F(x)$in (35), we get

$$F_{X_{(k)}}(x) = \sum_{\mathrm{i=k}}^{\mathrm{n}} \binom{n}{i} \sum_{j=0}^{n-j} \binom{n-i}{j} (-1)^j \left[\left(1+\frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right) e^{-\theta x^{-\alpha}}\right]^{\beta(i+j)}$$
$$F_{X_{(k)}}(x) = \sum_{i=k}^{n} \binom{n}{i} \sum_{j=0}^{n-j} \binom{n-j}{j} (-1)^j \sum_{l=0}^{\infty} \binom{\beta(i+j)}{j}$$
$$\times \frac{(\theta x^{-\alpha})^j}{(\theta^2+\theta+2)^j} (\theta x^{-\alpha}+\theta+2)^j e^{-j\theta x^{-\alpha}} \tag{36}$$

3.2 Maximum likelihood estimation of parameters of GIPS distribution

Let $\left(x_1, x_2, x_3, \ldots, x_n\right)$ be an independent identically distributed random samples of size n with probability density function defined in (11). Then, the likelihood function L of generalized inverse power Sujatha distribution is given by

$$L = \prod_{i=1}^{n} \frac{\beta \propto \theta^3}{\theta^2+\theta+2}(1 + x^{-\alpha} + x^{-2\alpha})x^{-(\alpha+1)}\left(\left(1 + \frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right)e^{-\theta x^{-\alpha}}\right)^{\beta-1} e^{-\theta x^{-\alpha}} \quad (37)$$

The log – likelihood function is given by $l = lnL(\alpha, \beta, \theta\ /\ x_1, x_2, \ldots, x_n)$

$$= \left\{\begin{array}{c} nln\beta + \mathrm{n}\ln\alpha + 3\,n\ln\theta - n\ln(\theta^2+\theta+2) \\ + \sum_{i=1}^{n} ln(1 + x^{-\alpha} + x^{-2\alpha}) - (\alpha+1)\sum_{i=1}^{n}\ln x - \theta\sum_{i=1}^{n}\ln x^{-\alpha} \\ +(\beta-1)\sum_{i=1}^{n} ln\,(1 + x^{-\alpha} + x^{-2\alpha})e^{-\theta x^{-\alpha}} \end{array}\right\} \quad (38)$$

Taking the first derivative of (38) with respect to β, α and θ, we get the following

$$\frac{dl}{d\beta} = \frac{n}{\beta} + \sum_{i=1}^{n} ln\left(1 + \frac{\theta x^{-\alpha}(\theta x^{-\alpha}+\theta+2)}{\theta^2+\theta+2}\right) - \theta\sum_{i=1}^{n} x^{-\alpha} = 0 \quad (39)$$

$$\frac{dl}{d\alpha} = \left\{\begin{array}{c} \frac{n}{\alpha} - \sum_{i=1}^{n}\left[\frac{x^{-\alpha}\ln x + 2x^{-2\alpha}\ln x}{1+x^{-\alpha}+x^{-2\alpha}}\right] - \sum_{i=1}^{n}\ln x \\ -2\beta\sum_{i=1}^{n}\left[\frac{\theta^2 x^{-2\alpha}\ln x + \theta x^{-\alpha}\ln x}{(\theta^2+\theta+2)(\theta^2+\theta+2+\theta^2 x^{-2\alpha}+2\theta x^{-\alpha})}\right] + \beta\theta\sum_{i=1}^{n} x^{-\alpha}\ln x \\ +2\sum_{i=1}^{n}\left[\frac{\theta^2 x^{-2\alpha}\ln x + \theta x^{-\alpha}\ln x}{(\theta^2+\theta+2)(\theta^2+\theta+2+\theta^2 x^{-2\alpha}+2\theta x^{-\alpha})}\right] - \theta\sum_{i=1}^{n} x^{-\alpha}\ln x \end{array}\right\} = 0 \quad (40)$$

$$\frac{\partial l}{\partial\theta} = \left\{\begin{array}{c} \frac{3n}{\theta} + \frac{n(2\theta+1)}{\theta^2+\theta+2} - \sum_{i=1}^{n}\ln x^{-\alpha} \\ -\beta\sum_{i=1}^{n}\left[\frac{\theta^2 x^{-2\alpha}+2\theta^3 x^{-\alpha}+2\theta^2 x^{-\alpha}+4\theta x^{-2\alpha}+4x^{-\alpha}}{(\theta^2+\theta+2)(\theta^2+\theta+2+\theta^2 x^{-2\alpha}+\theta^2 x^{-\alpha}+2\theta x^{-\alpha})}\right] - \beta\sum_{i=1}^{n} x^{-\alpha} \\ +\sum_{i=1}^{n}\left[\frac{\theta^2 x^{-2\alpha}+2\theta^3 x^{-\alpha}+2\theta^2 x^{-\alpha}+4\theta x^{-2\alpha}+4x^{-\alpha}}{(\theta^2+\theta+2)(\theta^2+\theta+2+\theta^2 x^{-2\alpha}+\theta^2 x^{-\alpha}+2\theta x^{-\alpha})}\right] + \sum_{i=1}^{n} x^{-\alpha} \end{array}\right\} = 0 \quad (41)$$

The maximum likelihood estimates of the parameters $\alpha, \beta\ and\ \theta$ are obtained by solving the nonlinear equations (39), (40) and (41) at

$$\frac{\partial l}{\partial\beta} = 0,\ \frac{\partial l}{d\alpha} = 0 \text{ and } \frac{\partial l}{\partial\theta} = 0$$

The interval estimation of any of the parameters of the GIPS distribution is possible when the necessary standard error estimate is known. As $n \to \infty$, the maximum likelihood $\widehat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\theta})$ of $\Theta = (\alpha, \beta, \theta)$ is asymptotically normally distributed with mean Θ and variance – covariance matrix.

$$v = \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix} = \begin{pmatrix} B_{11} & \beta_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & \beta_{33} \end{pmatrix}^{-1}$$

Where

$$\beta_{11} = \frac{\partial^2 l}{\partial\beta^2}, \quad \beta_{12} = \frac{-\partial^2 l}{\partial\beta\partial\gamma_1}, \quad \beta_{13} = \frac{\partial^2 l}{\partial\beta\partial\theta}$$

$$\beta_{22} = \frac{-\partial^2 l}{\partial\beta^2}, \quad \beta_{23} = \frac{-\partial^2 l}{\partial\alpha\partial\theta}, \quad \beta_{33} = \frac{-\partial^2 l}{\partial\theta^2}$$

Consequently, an appropriate 100 (1 – α)% confidence intervals for α, β and θ are defined as follows:

$$\hat{\beta} \pm z_{\frac{\alpha}{2}}\sqrt{\hat{v}_{11}}\ ,\ \hat{\alpha} \pm z_{\frac{\alpha}{2}}\sqrt{\hat{v}_{22}} \text{ and } \hat{\theta} \pm z_{\frac{\alpha}{2}}\sqrt{\hat{v}_{33}} \quad (42)$$

Hence, we use the R package in obtaining the parameter estimates and their corresponding standard error .

4 Numerical Applications of Generalized Inverse Power Sujatha Distribution

The elementary objective in this section is to encourage the use of the generalized inverse power Sujatha distribution. Thus, we have shown below, a successful applications to modelling two lifetime data sets given in Rameesa et al. [24] and Efron [25] which respectively represent the active repair times (hours) for an airborne communication transceiver and the survival times of a group of patients suffering from head and neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT). The first and second data sets are as follows:

Data I

0.50, 0.60, 0.60, 0.70, 0.70, 0.70, 0.80, 0.80,1.00, 1.00, 1.00, 1.00, 1.10, 1.30, 1.50, 1.50,1.50, 1.50, 2.00, 2.00, 2.20, 2.50, 2.70, 3.00,3.00, 3.30, 4.00, 4.00, 4.50, 4.70, 5.00, 5.40,5.40, 7.00, 7.50, 8.80, 9.00, 10.20, 22.00, 24.50

Data II

12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36,63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127,130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281,319, 339, 432, 469, 519, 633, 725, 817, 1776

5 Results and Discussion

Tables 1and 2 below shows the results obtained for Generalized Inverse Power Sujatha distribution (GIPS), Sujatha distribution (SD), Inverse Power Sujatha distribution (IPS) and Marshall–Olkin Extended Inverse Pareto Distribution (MOEIP) using datasets (I) and (II) in R software.

The estimation of the parameters of each of the distributions is made using the maximum likelihood method of estimation. The standard errors of each of the parameters were also estimated. The performance measures obtained include Log-likelihood (LL), Akaike's information criterion (AIC), Kolmogorov-smirnov (K-S) statistic, and the corresponding probability value (p-value). Goodness of fit has been decided using Akaike information criteria (AIC), Bayesian Information and criteria (BIC) values respectively, which are calculated for each distribution and also compared. The best distribution was decided on the basis of minimum value of AIC and BIC.

Giving to the lesser values of both AIC and BIC, the proposed distribution performs better than the other candidate models in both data sets. However, the largest negative LL value for the GIPS distribution, we infer that the latter offers a good fit for the given data and it proves thus to be the more appropriate model.

Tables 3 and 4 below show the 95% confidence interval constructed for the parameters of the GIPS distribution, using the first and second datasets respectively. The 95% confidence interval was estimated using (42).

From Tables 3 and 4, a 95% confidence interval was constructed for the parameters of each of the distribution using data sets I and II. The values of the upper and lower limits for the parameters indicated thatthe true values of the parameters lies within the confidence limits.

Table 1. MLE's, -LL, AIC, BIC, KS and p value of the fitted distributions using dataset (I)

Model	Parameters	S.E	LL	AIC	BIC	KS	p
GIPS							
	β =0.9662	**3.8442**	**-89.3234**	**184.6469**	**189.7135**	**0.096**	**0.8202**
	α=1.1620	**0.2308**					
	θ=2.5689	**6.9283**					
SD	θ = 0.6304	0.0572	-105.2646	212.5292	214.2181	0.2723	0.004127
IPS	α= 1.0961	0.181	-118.9097	241.8194	245.1971	0.7005	8.88E-16
	θ = 0.7174	0.112					
MOEIP	α = 84.7141	158.9557	-89.938	185.8759	190.9425	0.9808	8.88E-16
	β= 0.0252	0.0484					
	θ= 0.5100	0.2575					

Table 2. MLE's, -LL, AIC, BIC, KS and p value of the fitted distributions using dataset (II)

Model	Parameters	S.E	LL	AIC	BIC	KS	p
GIPS	**β =6.945384**	**96.01152**					
	α=1.013006	**0.111578**	**-279.565**	**565.1301**	**570.4827**	**0.0924**	**0.813**
	θ=12.682108	**159.9203**					
SD	θ = 0.01339386	0.0011592	-304.6933	611.3865	613.1707	0.27923	0.001587
IPS	α = 0.1845318	0.0307711	-385.3383	774.6766	778.245	0.88156	6.66E-16
	θ = 0.8528411	0.1363627					

Table 3. MLEs of the parameters GIPS distribution and their C.I (DATA I)

Model	parameter	S.E	95% Confidence Interval	
			Lower Limit	Upper Limit
GIPS	α=1.1620115	0.2307621	0.7097	1.6143
	β = 0.9661717	3.8441628	-6.5684	8.5007
	θ = 2.5689330	6.9282512	-11.0104	16.1483
SD	θ=0.6303782	0.05715657	0.5184	0.7424
IPS	α = 1.0960977	0.1809886	0.7414	1.4508
	θ = 0.7174331	0.1120384	0.4978	0.937
MOEIP	α = 84.71407192	158.9556752	-226.8391	396.2672
	β= 0.02519234	0.04837699	-0.0696	0.12
	θ = 0.50996972	0.25751662	0.0052	1.0147

Table 4. MLEs of the parameters GIPS distribution and their C.I (DATA II)

Model	parameter	S.E	95% Confidence Interval	
			Lower Limit	Upper Limit
GIPS	β =6.945384	96.011517	-181.2372	195.1279
	α=1.013006	0.1115782	0.7943	1.2317
	θ=12.682108	159.92027	-300.7616	326.1258
SD	θ=0.6303782	0.05715657	0.5184	0.7424
IPS	α = 1.0960977	0.1809886	0.7414	1.4508
	θ = 0.7174331	0.1120384	0.4978	0.937

6 Conclusion

In this paper, we proposed a new distribution known as the GIPS distribution. The distribution was formulated using the exponentiated method of generating new distributions. We also provided the mathematical characteristics of the new distribution including reliability functions such as survival function, hazard function, cumulative hazard function, reverse hazard rate and odds functions. We derived the maximum likelihood estimation of the parameters as well as the 95% confidence interval for the parameter estimates. To demonstrate the potential of the GIPS distribution, we considered two lifetime data sets and compared the goodness of fit with other famous probability distributions. The proposed distribution excelled the competitive models.

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The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

Competing Interests

Authors have declared that no competing interests exist.

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