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Research Article

Critical Phenomena of Charged AdS Black Holes in Rastall Gravity

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We construct analytical charged anti-de Sitter (AdS) black holes surrounded by perfect fluids in four dimensional Rastall gravity. Then, we discuss the thermodynamics and phase transitions of charged AdS black holes immersed in regular matter like dust and radiation, or exotic matter like quintessence, ΛCDM type, and phantom fields. Surrounded by phantom field, the charged AdS black hole demonstrates a new phenomenon of reentrant phase transition (RPT) when the parameters Q, N_p, and ψ satisfy some certain condition, along with the usual small/large black hole (SBH/LBH) phase transition for the surrounding dust, radiation, quintessence, and cosmological constant fields.

1. Introduction

As well known, Einstein's theory of general relativity (GR) has made a lot of interesting predictions for solar system experiments, which are in perfect agreement with the observational data. On larger (cosmological) scales, there are lines of strong observational evidence implying that dark matter (DM) and dark energy (DE) account for about 95% of the universe [1, 2]. From these observations, many theoretical models have been presented in Refs. [3–5]. In 1972, one of the potential gravity theories was proposed by Rastall [6, 7]. In the Rastall gravity theory, the usual conservation law of the energy–momentum tensor $\left(T^{\mu\nu}{}_{;\mu} = 0\right)$ is not obeyed, and the energy–momentum tensor satisfies

$$T^{\mu\nu}{}_{;\mu} = \lambda R^{\nu}, \tag{1}$$

where R is the Ricci scalar, and the parameter λ measures the potential deviations of Rastall theory from GR. It is interesting that all electrovacuum solutions in GR are also solutions of Rastall gravity. However, if matter field (nonvanishing trace) is introduced, the spacetime depends on the Rastall parameter λ. As the simplest modified gravity scenario, it provides an explanation of the inflation problem, and realizes the late-time acceleration and other cosmological problems [8–11]. Until now, many works on the various black hole solutions have been also investigated in Rastall theory. The spherically symmetric black hole solutions were constructed in Refs. [12–16], the rotating black holes in Refs. [17, 18], the thermodynamics of black holes in Refs. [19–22], and also instability of black holes in Refs. [23, 24].

According to the AdS/CFT correspondence, the bulk AdS black hole spacetimes admit a gauge duality description by thermal conformal field theory living on the AdS boundary [25–27]. For example, the usual Hawking-Page phase transition [28] in four dimensional GR can be interpreted as a confinement/deconfinement phase transition in the dual quark gluon plasma [29]. Recently, the study of thermodynamics of AdS black holes has been extended to the extended phase space, where the cosmological constant is treated as the pressure of the black hole [30, 31]. In the extended phase space, charged AdS black hole admits a more direct and precise coincidence between the first order small/large black holes (SBH/LBH) phase transition and Van der Waals (VdW) liquid–gas phase transition [32], and both systems share the same critical exponents near the critical point. More discussions in this direction can be found as well in Refs. [33–61], including reentrant phase transitions and some other phase transitions. In the Rastall theory, Ali recovered the existence of VdW like

Table 1: The condition for $\rho_s \geq 0$.

$N_s > 0 \cup W_s < 0$	$\omega_s > -1 \cup -2 < \xi < 1$	$\omega_s < -1 \cup \xi > 1$	$\omega_s < -1 \cup \xi < -2$
$N_s < 0 \cup W_s > 0$	$\omega_s < -1 \cup -2 < \xi < 1$	$\omega_s > -1 \cup \xi > 1$	$\omega_s > -1 \cup \xi < -2$

SBH/LBH phase transition for AdS black holes in four dimensional spacetimes [62]. Motivated by this result, it is interesting to explore the effect of charge Q on the critical behaviors for charged AdS black holes in four dimensional Rastall theory. We hope to reveal a more fascinating phenomenon.

This paper is organized as follows. In Section 2, the analytical solution for static spherical symmetric charged AdS black holes are obtained in four dimensional Rastall theory. In Section 3, we study the critical behaviors of the charged AdS black holes in context of $P-V$ criticality and phase diagrams. We end the paper with conclusion and discussion in Section 4.

2. Charged AdS Black Hole Solutions in Rastall Theory

Considering Equation (1), the field equation including the negative cosmological constant Λ can be written as [62]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \kappa\lambda g_{\mu\nu} R = T_{\mu\nu}, \tag{2}$$

where κ equals to $8\pi G_N$ and G_N is the Newton gravitational coupling constant.

In order to derive the black hole solutions, we consider the general spherical symmetric spacetime metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\vartheta^2\right), \tag{3}$$

where $f(r)$ is a generic metric function depending on the radial coordinate r. Using this metric, the nonvanishing components of right hand side of Equation (2) are

$$\begin{aligned} H_0^0 &= H_1^1 = G_0^0 + \Lambda + \kappa\lambda R = \frac{1}{r^2}\left(f'r - 1 + f\right) + \Lambda + \kappa\lambda R, \\ H_2^2 &= H_3^3 = G_2^2 + \Lambda + \kappa\lambda R = \frac{1}{r^2}\left(rf' + \frac{1}{2}r^2 f''\right) + \Lambda + \kappa\lambda R, \end{aligned} \tag{4}$$

with the Ricci scalar $R = -\left(1/r^2\right)\left(r^2 f'' + 4rf' - 2 + 2f\right)$.

On the other hand, the total energy–momentum tensor $T_{\mu\nu}$ supporting this spacetime takes the following form [13, 14]

$$T^{\mu}{}_{\nu} = E^{\mu}{}_{\nu} + \mathcal{T}^{\mu}{}_{\nu}, \tag{5}$$

where E^{μ}_{ν} is given by

$$E_{\mu\nu} = \frac{2}{\kappa}\left(F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{6}$$

and $F_{\mu\nu}$ satisfies the Maxwell equations $F^{\mu\nu}{}_{;\nu} = 0$. Using this Maxwell equation, we can derive

$$F^{01} = \frac{Q}{r^2}, \tag{7}$$

where Q is an integration constant playing the role of a electrostatic charge. Then the nonvanishing components of the Maxwell tensor $E^{\mu}{}_{\nu}$ read as

$$E^{\mu}{}_{\nu} = \frac{Q^2}{\kappa r^4}\mathrm{diag}(-1,-1,1,1). \tag{8}$$

In addition, $\mathcal{T}^{\mu}{}_{\nu}$ describes the energy–momentum tensor of surrounding field defined as [63]

$$\begin{aligned} \mathcal{T}^{0}{}_{0} &= \mathcal{T}^{1}{}_{1} = -\rho_s(r), \\ \mathcal{T}^{2}{}_{2} &= \mathcal{T}^{3}{}_{3} = \frac{1}{2}\left(1 + 3\omega_s\right)\rho_s(r), \end{aligned} \tag{9}$$

where $\rho_s(r)$ is the energy density and satisfies barotropic equation of state $p_s = \omega_s\rho_s$ with the pressure $p_s(r)$ and state parameter ω_s. The subscript "s" denotes the surrounding field which can be a dust ($\omega_d = 0$), radiation ($\omega_r = 1/3$), ΛCDM type ($\omega_\Lambda = -1$), quintessence and phantom field constructed by the combination of these fields.

From $H_0^0 = T_0^0$ and $H_2^2 = T_2^2$ components of Rastall field equations, the solution of four dimensional charged AdS black hole can be obtained as

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\vartheta^2\right), \\ f(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{N_s}{r^{\xi}} + \frac{r^2}{l_\Lambda^2}, \end{aligned} \tag{10}$$

where M and N_s are two integration constants representing the black hole mass and surrounding field structure parameter, respectively. The parameters ξ and l_Λ are given by

$$\begin{aligned} \xi &= \frac{1 + 3\omega_s - 6\psi(1 + \omega_s)}{1 - 3\psi(1 + \omega_s)}, \\ \frac{1}{l_\Lambda^2} &= -\frac{\Lambda}{3(1 - 4\psi)}, \quad \psi \equiv \kappa\lambda, \end{aligned} \tag{11}$$

where l_Λ is the curvature radius, and N_s is related to the energy density ρ_s

$$\rho_s = -\frac{3W_s N_s}{\kappa r^{\left(3(1+\omega_s) - 12\psi(1+\omega_s)\right)/\left(1 - 3\psi(1+\omega_s)\right)}}, \tag{12}$$

where W_s is

$$W_s = -\frac{(1 - 4\psi)\left(\psi(1 + \omega_s) - \omega_s\right)}{\left(1 - 3\psi(1 + \omega_s)\right)^2}. \tag{13}$$

Regarding the weak energy condition representing the positivity of any kind of energy density of the surrounding field, i.e., $\rho_s \geq 0$, imposes the following conditions

$$W_s N_s \leq 0, \tag{14}$$

which implies that for the surrounding field with $W_s > 0$, we need $N_s < 0$ and conversely for $W_s < 0$, we need $N_s > 0$. If $\omega_s \neq -1$, the constraint conditions for ω_s and ξ can be derived, see Table 1. For the cosmological constant field $\omega_s = \omega_\Lambda = -1$, ξ equals to -2 which is independent of ψ and W_Λ is $-1 + 4\psi$. Then, we find

$$N_\Lambda > 0, \quad 0 < \psi < \frac{1}{4}, \text{or} \quad N_\Lambda < 0, \quad \psi > \frac{1}{4} \tag{15}$$

from the weak energy condition Equation (11).

TABLE 2: Real root r_c of Equation (23).

r_c	$\omega_s > -1$		$\omega_s < -1$	
	$N_s > 0$	$N_s < 0$	$N_s > 0$	$N_s < 0$
$\xi < -2$	None	1	0,1,2	None
$-2 < \xi < -1$	1	None	None	0,1,2
$-1 < \xi < 0$	0,1,2	None	None	1
$0 < \xi \leq 2$		1		
$2 < \xi$	None	1	0,1,2	None

3. Critical Behaviors of Charged AdS Black Holes

In the extended phase space, we interpret the cosmological constant Λ as a positive thermodynamic pressure P in the geometric units $G_N = \hbar = c = 1$

$$P = -\frac{\Lambda}{8\pi} = \frac{3(1-4\psi)}{8\pi l_\Lambda^2}. \tag{16}$$

In terms of horizon radius r_+, mass M, Hawking temperature T, entropy S, and electromagnetic potential Φ of the charged AdS black holes can be written as

$$M = \frac{1}{2}\left(\frac{Q^2}{r_+} + r_+ - \frac{N_s}{r_+^{\xi-1}} + \frac{8\pi P r_+^3}{3(1-4\psi)}\right), \quad S = \pi r_+^2,$$
$$T = \frac{1}{4\pi}\left(\frac{1}{r_+} + \frac{8\pi P r_+}{1-4\psi} - \frac{Q^2}{r_+^3} + \frac{(\xi-1)N_s}{r_+^{1+\xi}}\right), \quad \Phi = \frac{Q}{r_+}. \tag{17}$$

Here the black hole mass M should be considered as the enthalpy rather than the internal energy of the gravitational system. Moreover, those thermodynamic quantities should satisfy first law of black hole thermodynamics

$$dM = TdS + VdP + \Theta_s dN_s + \Phi dQ, \tag{18}$$

where the thermodynamic volume V conjugate to P equals to $4\pi r_+^3/3(1-4\psi)$ and Θ_s as a generalized force conjugate to the parameter N_s reads as $-r_+^{1-\xi}/2$.

Using Equation (17), the equation of state $P(V,T)$ can be written as

$$P = \frac{(1-4\psi)}{2r_+}\left[T - \frac{1}{4\pi r_+} + \frac{Q^2}{4\pi r_+^3} - \frac{(\xi-1)N_s}{4\pi r_+^{1+\xi}}\right]. \tag{19}$$

As usual, a critical point occurs when P has an inflection point

$$\left.\frac{\partial P}{\partial r_+}\right|_{T=T_c, r_+=r_c} = \left.\frac{\partial^2 P}{\partial r_+^2}\right|_{T=T_c, r_+=r_c} = 0, \tag{20}$$

where the subscript "c" denotes the values of the physical quantities at the critical point. When $\omega_s \neq -1$, we can obtain

$$T_c = \frac{\xi r_c^2 - 2Q^2(\xi-2)}{2\pi(1+\xi)r_c^3}, \tag{21}$$

$$P_c = \frac{\left(\xi r_c^2 - 3Q^2(\xi-2)\right)(3\omega_s - 1)}{24\pi r_c^4(\xi-2)(\omega_s+1)}, \tag{22}$$

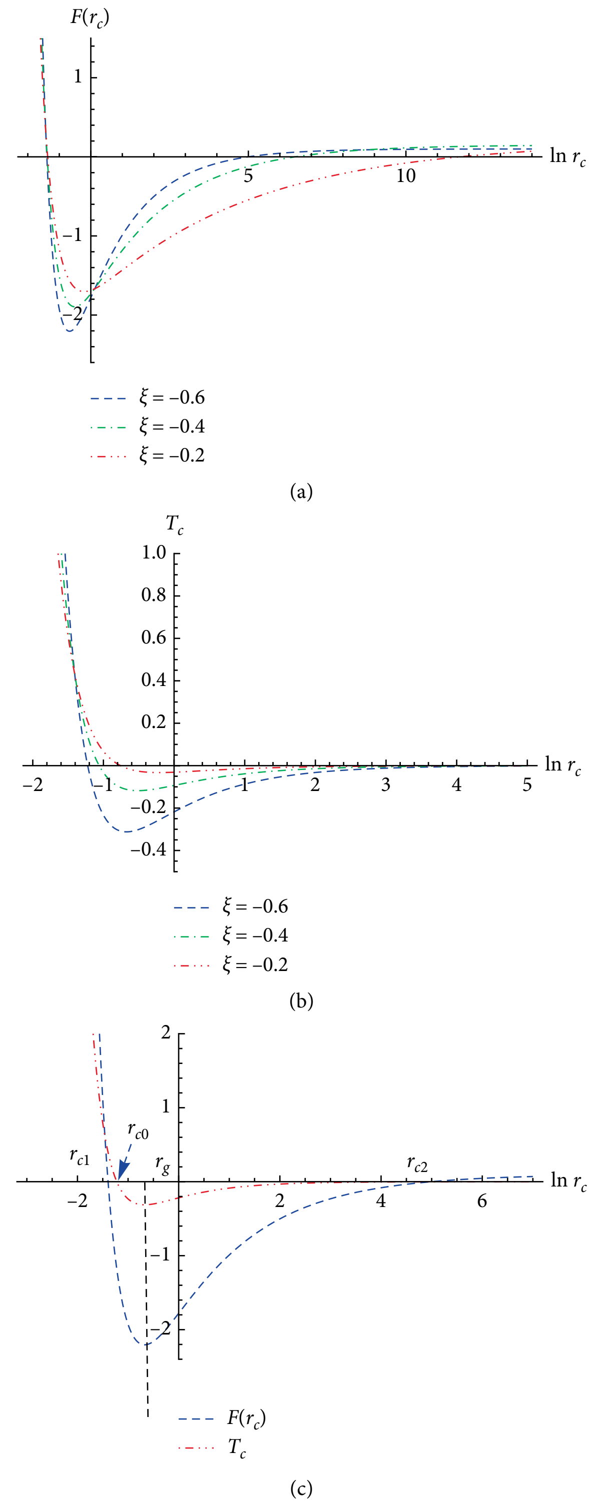


FIGURE 1: The $F(r_c) - \ln r_c$ and $T_c - \ln r_c$ diagrams with $Q = 0.1$ and $N_s = 0.1113$ for different values of ξ.

and the equation for critical horizon radius r_c

$$F(r_c) = (1-\xi)(1+\xi)(2+\xi)N_s - 2r_c^{\xi-2}\left(r_c^2 - 6Q^2\right) = 0 \tag{23}$$

with Equation (11). Evidently, it is hard to derive any analytic solution for the critical quantities r_c, T_c, and P_c from Equations (21)–(23). By analyzing the asymptotical behavior of function $F(r_c)$ at the horizon and infinity, we summarize the number of real root r_c of Equation (23) in Table 2. The discussion for cosmological constant field $\omega_s = -1$ will be shown in the last part of this section.

Taking the region of $\omega_s > -1$, $-1 < \xi < 0$ and $N_s > 0$ for instance, the function $F(r_c)$ [Equation (23)] approaches $+\infty$ near the origin $r \to 0$ and tends to $-N_s(1-\xi)(1+\xi)(2+\xi)(>0)$ when $r \to +\infty$. For the critical temperature T_c [Equation (21)], it approaches $+\infty$ near the origin $r \to 0$, T_c disappears at $r_c = r_{c0} = \sqrt{2(\xi-2)/\xi}Q$ and tends to 0 when $r \to +\infty$, see Figures 1(a) and 1(b). On the other hand, the derivative of $F(r_c)$, T_c, and P_c leads to a same expression

$$F'(r_c) = T_c' = P_c' = -6Q^2(-2+\xi) + r_c^2\xi, \tag{24}$$

which vanishes at $r_c \equiv r_g = \sqrt{6(\xi-2)/\xi}Q$ From $F(r_c) = 0$, the two real positive critical radius r_{c1} and r_{c2} (if exist) satisfy $r_{c1} < r_g < r_{c2}$. We have $T_c(r_{c2}) < 0$ because of $r_{c0} < r_g < r_{c2}$ and $T_c(r_{c1}) > 0$ if $r_{c1} < r_{c0}$, see Figure 1(c). In general, a black hole should possess positive temperature $T > 0$, and reduces to extremal one when $T = 0$. Moreover, holding positive temperature $T > 0$ makes it possible to set up an analogy between the realistic Van der Waals phase transition for liquid/gas system [64] and SBH/LBH phase transition for the charged AdS black hole in the Rastall gravity. For the four dimensional Reissner–Nordström AdS black hole, Kubiznak and Mann derived one set of positive critical quantities r_c, P_c, and T_c by utilizing Equation (20) in the extended phase space, and further recovered that Reissner–Nordström AdS black hole admits VdW-like SBH/LBH phase transition in the Einstein gravity [32].

Besides the VdW-like SBH/LBH phase transition, Ref. [48] firstly proposed the reentrant phase transition (RPT) for Born–Infeld AdS black hole in GR, if there exist more than one positive critical points. Nevertheless, the RPT does not happen in this region ($\omega_s > -1$, $-1 < \xi < 0$ and $N_s > 0$) for charged AdS black hole in Rastall gravity because of the unphysical critical temperature $T_c(r_{c2}) < 0$, even though there could exist two positive real critical radius r_c from Table 2.

In addition, to study the possible phase transitions in the system, we need to discuss the Gibbs free energy, which obeys the following thermodynamic relation $G = M - TS$ with

$$G = \frac{r_+}{4} + \frac{3Q^2}{4r_+} - \frac{2P\pi r_+^3}{3(1-4\psi)} - \frac{(1+\xi)N_s}{4r_+^{\xi-1}}, \tag{25}$$

where r_+ is understood as a function of pressure and temperature, $r_+ = r_+(P,T)$, via equation of state [Equation (19)]. In the rest of this section, we mainly focus on that regions possessing more than one real roots of r_c from Table 2, and then consider the critical behaviors of charged AdS black holes surrounded by dust, radiation, quintessence, phantom, or Λ CDM type fields.

3.1. Black Hole Surrounded by Dust or Quintessence Fields. For the dust surrounding field, we set $\omega_d = 0$. Taking $\xi = -0.8$, $Q = 0.1$ and $N_d = 0.1$ for instance, one can obtain two critical radius $r_{c1} = 0.245$ and $r_{c2} = 120.762$, and corresponding critical pressure and temperature $P_{c1} \approx 0.0463$, $T_{c1} \approx 0.4104$ and $P_{c2} \approx -2.5984 * 10^{-7}$, $T_{c2} \approx -0.00527166$. So, there only exists one physical critical point, which has been verified in the above part. As shown in Figure 2, $P - r_+$ diagram displays that the solid curve represents critical isotherm at $T = T_{c1}$ and the dashed and dotted curves correspond to $T > T_{c1}$ and $T < T_{c1}$, respectively. In the $G - T$ diagram, the solid curve represents $P < P_{c1}$, the dotted curve corresponds to $P > P_{c1}$, and the dashed curve is for $P = P_{c1}$. We observe standard swallowtail behavior. Moreover, the $P - T$ diagram shows the coexistence line of the first-order SBH/LBH phase transition terminating at a critical point. These plots are analogous to typical behavior of the liquid–gas phase transition of the VdW fluid.

We turn to discuss the quintessence field. The quintessential dark energy is governed by an equation of state of the form $p_q = \omega_q\rho_q$ with the state parameter $\omega_q \in (-1,-1/3)$ [63]. Here we choose a ordinary example $\omega_q = -2/3$ [63, 65]. Taking $\xi = -0.7$, $Q = 0.4$ and $N_q = 0.4$, one can obtain two critical radius $r_{c1} = 1.055$ and $r_{c2} = 17.85$, and corresponding critical pressure and temperature $P_{c1} \approx 0.0184$, $T_{c1} \approx 0.0382$ and $P_{c2} \approx -0.000096$, $T_{c2} \approx -0.0207$. Therefore, there only exists one physical critical point, which implies the appearance of VdW-like SBH/LBH phase transition. The $P - r_+$, $G - T$, and $P - T$ diagrams are similar with Figure 2, so we do not plot these figures in this case.

3.2. Black Hole Surrounded by Radiation Field. With regard to the case of $\omega_r = 1/3$, one have $\xi = 2$, which is independent of ψ from Equation (11). Then, the black hole solution (10) becomes

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2 - N_r}{r^2} + \frac{r^2}{l_\Lambda^2}, \tag{26}$$

Obviously, the parameter N_r plays a shift role in the charge Q. It's worthy to point out that the condition $Q^2 > N_r$ needs to be satisfied so that we can derive the possible phase transition. Then, the equation of state $P(V,T)$ reduces to

$$P = \frac{(1-4\psi)}{2r_+}\left(T - \frac{1}{4\pi r_+} + \frac{Q^2}{4\pi r_+^3} - \frac{N_r}{4\pi r_+^3}\right). \tag{27}$$

According the judgement condition (20), corresponding critical values read as

$$\begin{aligned} r_c &= \sqrt{6\left(Q^2 - N_r\right)}, \quad P_c = \frac{1-4\psi}{96\pi\left(Q^2 - N_r\right)}, \\ T_c &= \frac{1}{3\sqrt{6\left(Q^2 - N_r\right)}}, \quad Q^2 > N_r, \quad \psi < \frac{1}{4}. \end{aligned} \tag{28}$$

Then, VdW-like SBH/LBH phase transition occurs for charged AdS black hole in the region of $Q^2 > N_r$ and $\psi < 1/4$, which is similar as that for charged AdS black hole in Einstein gravity [32].

3.3. Black Hole Surrounded by Phantom Field. For the phantom field, it possesses the supernegative equation of state $\omega_p < -1$ [65]. Nevertheless, when a black hole immersed this phantom

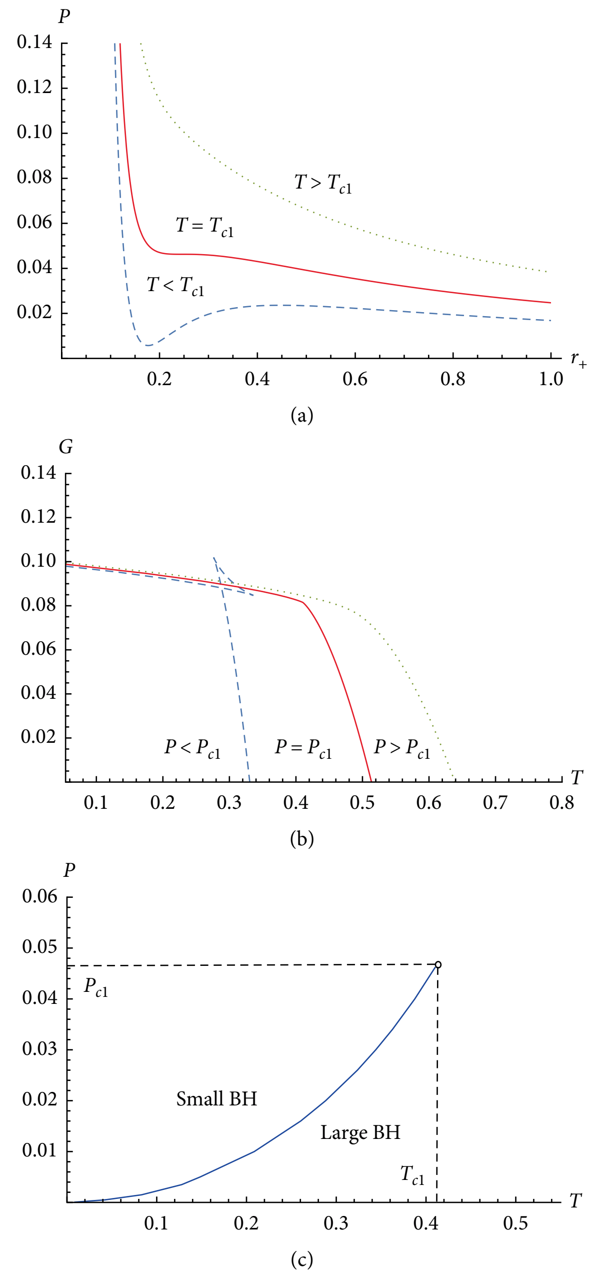


FIGURE 2: The $P - r_+$, $G - T$, and $P - T$ diagrams of four-dimensional charged AdS black holes with $\omega_d = 0, \xi = -0.8, Q = 0.1$, and $N_d = 0.1$. There is one critical point, which corresponds to VdW-like SBH/LBH phase transition when $T < T_{c1} \approx 0.4104$ and $P < P_{c1} \approx 0.0463$.

field, it gives rise to $p_p + \rho_p < 0$ and $T_p s_p < 0$, and leads to two alternatives: either the entropy density s_p is negative with positive temperature T_p or the entropy density s_p is positive but the temperature is negative. With $T_p > 0$, the entropy of black hole decreases as it accretes the phantom energy, meanwhile a decrease in the energy density of phantom field decreases the entropy of phantom field [66, 67]. Finally, the total entropy of thermodynamic system consisting of phantom field entropy and black holes entropy will decrease simultaneously, which violates the generalized second law (GSL) of thermodynamics, and then the accretion process could never occur in this case.

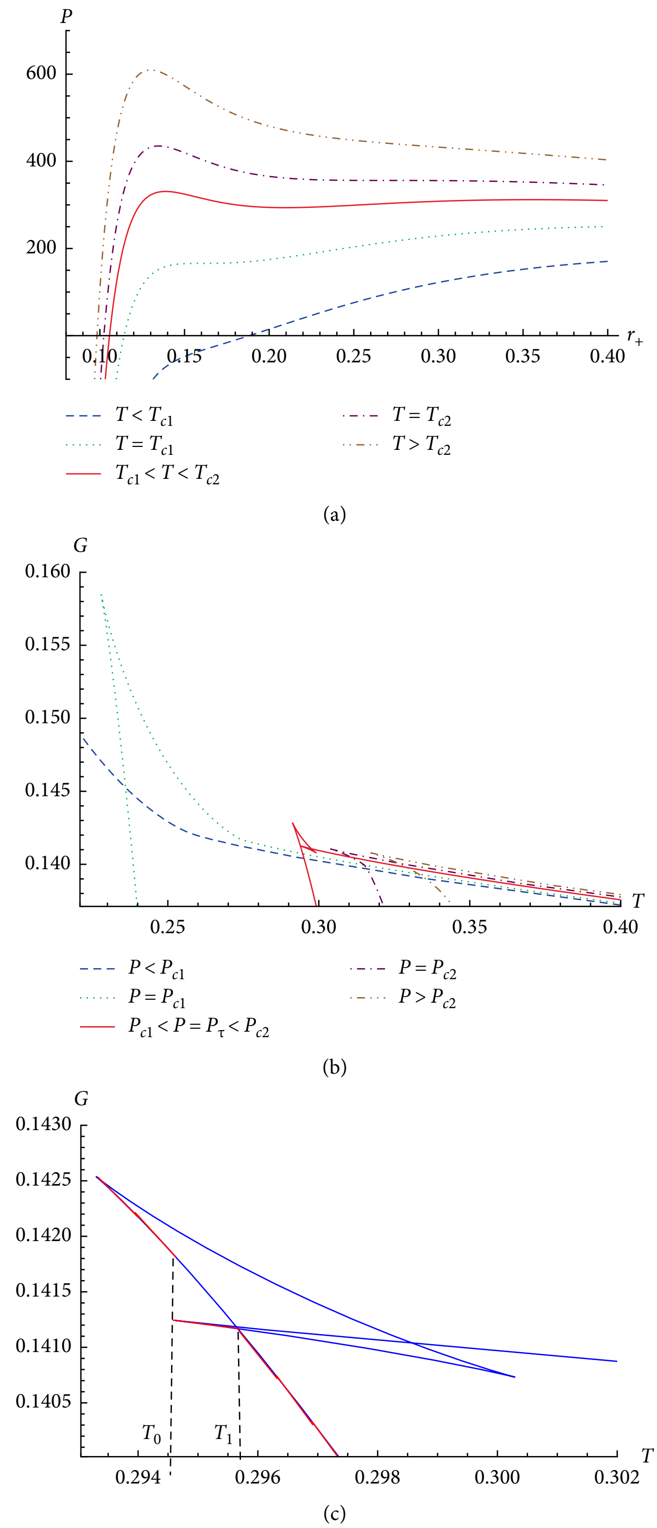
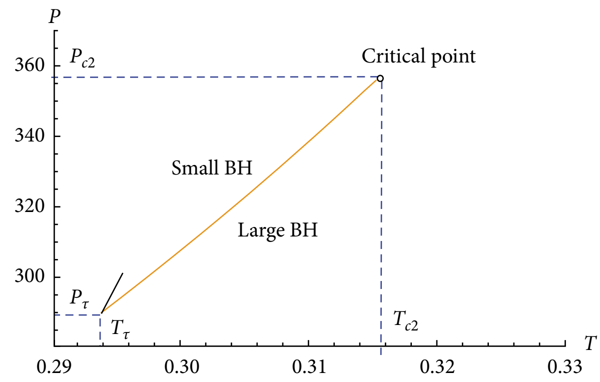
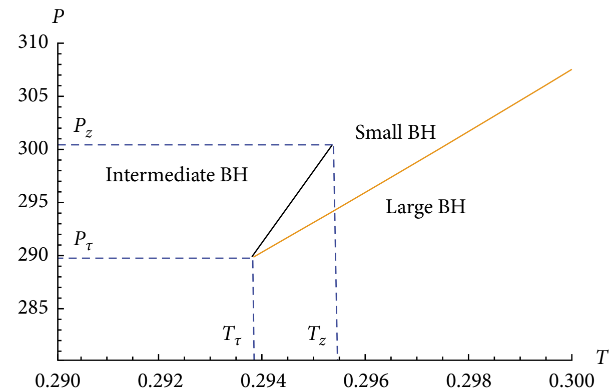


FIGURE 3: The $P - r_+$ and $G - T$ diagrams of four-dimensional charged AdS black holes with $\omega_p = -1.03$, $\xi = 2.1$, $Q = 0.4$, and $N_p = 0.1113$. It shows the existence of two critical points $(P_{c1} = 165.8340, T_{c1} = 0.2743)$ and $(P_{c2} = 356.10, T_{c2} = 0.3153)$. For $P \in (P_\tau, P_z)$, we observe a "zeroth-order phase transition" at T_0 signifying the onset of an RPT in (c).

In terms of $T_p < 0$, the accretion of phantom field by this black hole is also not possible if the mass of black hole is above



(a)



(b)

FIGURE 4: The $P-T$ diagram of four-dimensional charged AdS black holes with $\omega_p=-1.03, \xi=2.1, Q=0.4$, and $N_p=0.1113$. The thick solid line initiating from (P_{c2},T_{c2}) and terminates at (P_τ,T_τ) corresponds to the co-existence line of VdW-like SBH/LBH phase transition. The solid line commencing from (P_z,T_z) and terminates at (P_τ,T_τ) denotes the coexistence line of small and intermediate black holes indicating the RPT.

from corresponding critical value [68–71]. Here we only discuss thermodynamics and phase transitions of charged AdS black holes in the Rastall gravity. Therefore, we can assume the phantom field with $T_p>0$ for simplicity, so that the thermodynamic system consisting of charged AdS black hole and phantom field is always stable.

Recently, Kumar and Xu adopted the observational data sets of SNLS3, BAO, *Planck* + WMAP9 + WiggleZ to determine the parameter $\omega_p=-1.06^{+0.11}_{-0.13}$ [72]. Moreover, the Planck 2018 results recovers $\omega_p=-1.03\pm0.03$ [73]. In view of these constraints on ω_p from observations, we consider the phantom field with $\omega_p=-1.03$. With $N_p>0$ and $\xi>2$ in Table 2, the function $F(r_c)$ [Equation (23)] tends to $-N_p(\xi-1)(\xi+1)(\xi+2)(<0)$ as $r_c\to0$ and $-\infty$ when $r_c\to-\infty$. Based on $F(r_c)=0$ and Equations (21) and (22), we can obtain two positive critical points when the parameters Q, ξ, and N_p satisfy

$$\frac{2\left(\sqrt{3(\xi-2)/\xi}Q\right)^{\xi}}{(\xi-2)(\xi-1)(\xi+1)}<N_s<\frac{4\left(\sqrt{6(\xi-2)/\xi}Q\right)^{\xi}}{(\xi-2)(\xi-1)(\xi+1)(\xi+2)}. \tag{29}$$

By using the graphical and numerical method, $P-r_+$ and $G-T$ diagrams of charged AdS black hole with $\xi=2.1$, $Q=0.4$, and $N_p=0.1113$ are displayed in Figure 3. One can find two physical critical points [Figure 3(a)]. For $P<P_{c2}(T<T_{c2})$, the VdW-like SBH/LBH phase transition occurs, and terminates at $P=P_\tau(T=T_\tau)$. At the same time, there also exists a certain range of $P\in(P_\tau,P_z)$, namely $T\in(T_\tau,T_z)$, for which the global minimum of G is discontinuous; see Figure 3(c). In this range of temperatures, two separate branches of intermediate size and small size black holes co-exist. They are separated by a finite jump in G, which is so-called "zeroth-order phase transition".

This novel situation can be also clearly illustrated in the $P-T$ diagrams in Figure 4. There is the expected SBH/LBH line of co-existence, which initiates from the critical point (T_τ,P_τ) and terminates at (T_{c2},P_{c2}). Especially, a "triple point" between the small (SBH), intermediate (IBH), and large black holes (LBH) emerges in the point (T_τ,P_τ). In the region of $T\in(T_\tau,T_z)$, a new IBH/SBH line of coexistence appears and then it terminates in another critical point (T_z,P_z). These values are obtained as

$$\begin{aligned}(T_\tau,T_z,T_{c2})&\approx(0.2938,0.2954,0.3153),\\(P_\tau,P_z,P_{c2})&\approx(289.8545,300.2650,356.10).\end{aligned} \tag{30}$$

Considering the other regions of $N_p>0$, $\xi<-2$, or $N_p<0, -2<\xi<-1$ in Table 2, two positive roots of r_{c1} and r_{c2} could appear from equation $F(r_c)=0$. Taking $N_p>0$ and $\xi<-2$ for instance, we choose $\omega_p=-1.03, \xi=-2.4, Q=0.2$, and $N_p=0.4$, and find that there only exists the VdW-like SBH/LBH phase transition in the region of $T_{c1}<T_c<T_{c2}$ and $P_{c1}<P_c<P_{c2}$ with

$$(P_{c1},P_{c1})\approx(0.17836,0.43678),\quad(T_{c2},T_{c2})\approx(0.23295,0.57949), \tag{31}$$

see Figure 5. Similar discussion can be done in the region of $N_p<0, -2<\xi<-1$ and $\omega_p=-1.03$, we find there only is the VdW-like SBH/LBH phase transition, even though there also exist two different physical critical points: $(r_{c1},T_{c1},P_{c1})>0$ and $(r_{c2},T_{c2},P_{c2})>0$.

3.4. Black Hole Surrounded ΛCDM Type Field. In case of $\omega_\Lambda=-1$, namely $\xi=-2$, the critical horizon radius, temperature, and pressure are obtained as

$$r_c=\sqrt{6}Q,\quad T_c=\frac{1}{3\sqrt{6}\pi Q}\quad P_c=\frac{\left(1+36N_\Lambda Q^2\right)(1-4\psi)}{96\pi Q^2}. \tag{32}$$

Holding $P_c>0$, the parameters N_Λ and ψ should satisfy $N_\Lambda<-1/36Q^2$ for $\psi>1/4$ and $N_\Lambda>0$ for $0<\psi<1/4$ in terms of the condition (15). Finally, VdW-like SBH/LBH phase transition occurs in these regions, which is similar as that for charged AdS black hole in Einstein gravity [32].

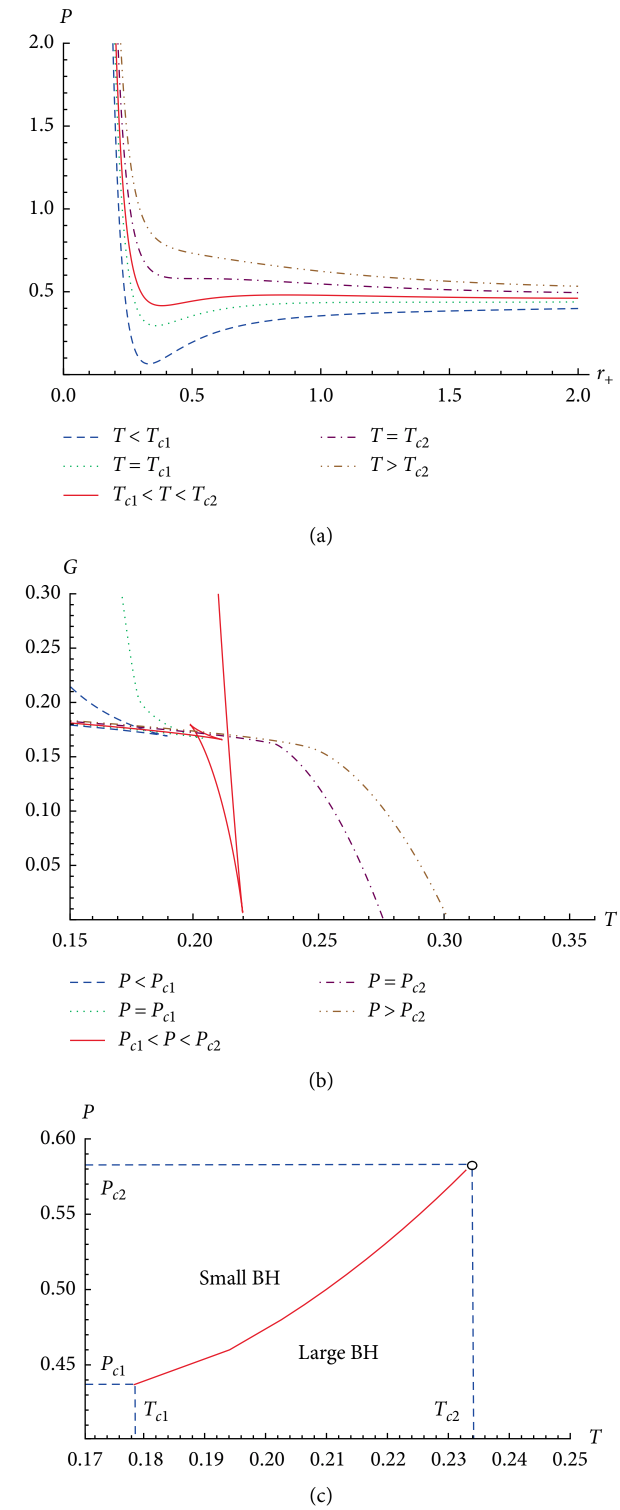


FIGURE 5: The $P-r_+$, $G-T$, and $P-T$ diagrams of four-dimensional charged AdS black holes with $\omega_p = -1.03$, $\xi = -2.4$, $Q = 0.2$, and $N_p = 0.4$. The VdW-like SBH/LBH phase transition happens in the region of $T_{c1} < T < T_{c2}$ and $P_{c1} < P < P_{c2}$.

4. Conclusion and Discussion

We have obtained charged AdS black holes solutions surrounded by perfect fluid in the context of four dimensional Rastall theory. Regarding the weak energy condition of energy density and asymptotic behavior of function $F(r_c)$, we derived the constraint conditions for the parameters N_s, ω_s and ξ or ψ, as shown in Tables 1 and 2. Later, we chose some special surrounding fields which can be a dust, radiation, quintessence, Λ CDM type and phantom field, and found that the Van der Waals like SBH/LBH phase transition happens for the charged AdS black holes with dust, radiation or quintessence and Λ CDM type fields. In particular, the charged AdS black hole surrounded by phantom field showed more interesting phenomenon in the region of $N_p > 0$ and $\xi > 2$. When the parameters Q, ξ, and N_p satisfy $\left(2\left(\sqrt{3(\xi-2)/\xi}Q\right)^{\xi}/(\xi-2)(\xi-1)(\xi+1)\right) < N_p < \left(4\left(\sqrt{6(\xi-2)/\xi}Q\right)^{\xi}/(\xi-2)(\xi-1)(\xi+1)(\xi+2)\right)$ and $\xi > 2$, we obtained two positive critical points and so-called RPT phase transition occurs, besides the VdW-like SBH/LBH phase transition. However, in the region of $N_p > 0$, $\xi < -2$, or $N_p < 0$, $-2 < \xi < -1$, there only exist the VdW-like SBH/LBH phase transition, even though it has two physical critical points.

Comparing with the Maxwell field, the Born–Infeld AdS black hole, and charged AdS black hole with Yang–Mills field in the Einstein gravity have proposed more interesting thermodynamical properties. In the Rastall gravity, one can also construct new black holes with nonlinear electrodynamics, and then explore whether the Van der Waals or reentrant phase transition can appear.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

The supplementary files are data for the Figures 2(b) and 2(c); Figures 3(b) and 3(c), Figure 4, and Figures 5(b) and 5(c). (*Supplementary Materials*)

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