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The Formation of 6 and 8 Members Block Hybrid Methods with Multiple-mesh Points for the Solution of Stiff Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author YS designed the methods, performed the statistical analysis, wrote the protocol and first draft of the manuscript. Authors JS and HA managed the analysis of the study. Author HA managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

This paper is focused on the formation of 6 and 8 member block hybrid methods with multiple-mesh point's for the solution of stiff ordinary differential equations. The research further investigates the basic properties of implicit three-step and four-step block hybrid methods. We noticed that the moment the value of an error constant is positive, the order P is odd. And when the value of an error constant is negative then the order P is of even number. The performance of the methods was demonstrated on some stiff initial value problems (IVPs). The result revealed that the hybrid block methods are efficient, accurate and convergent on some stiff ordinary differential equations.

Keywords: Implicit; hybrid block method; BHM; stiff ODEs.

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1. INTRODUCTION

The hybrid methods have been developed since the 1960's but these methods have not yet received a great deal of attention, see [1,2,3] in the literature, despite their higher accuracy over the single linear multi-step methods of the same step size k. The main reason for this may be due to the fact that, the need for special predictors to estimate the off-step solutions present in the corrector formulae.

Following Onumanyi *et-al* [4, 5] we identify a continuous hybrid method through the addition of one or more off mesh collocation points in the multi-step collocation. The single continuous hybrid scheme is evaluated at some distinct points involving mesh and off-mesh points along with its first derivative, where necessary, to give multiple hybrid block methods for the treatment of stiff ordinary differential equations.

This paper is classified into sections. In section 2.0 the multi-step collocation procedure is constructed involving multiple off-grid collocation points. In section 3.0, we obtain the order and error constants in a block form, the stability regions are also plotted, the numerical implementation of the block hybrid schemes on stiff (ODEs) and conclusion is given in section 4.0. We consider the numerical solution of first order initial value problems of the form: $y' = f(x,y), \quad y(x_0) = y_0$

Where f is continuous and satisfies Lipchitz's condition that guarantees the uniqueness and existence of a solution.

2. CONSTRUCTIONS OF THE METHODS

2.1 Derivation Techniques of MC [1, 2, 3, 4, 5]

Consider the collocation methods defined by

$$y(x) = \sum_{j=0}^{t-1} \alpha_j(x)(y_{n+1}) + h\sum_{j=0}^{m-1} \beta_j(x) f\left(x_j, y\left(\bar{x}_j\right)\right)$$

and also we get D matrix as follows:

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & \dots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & \dots & x_n^{t+m-1} \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & \dots & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0 & \dots & (t+m-1)x_0^{-t+m-2} \\ 0 & 1 & 2\bar{x}_1 & \dots & (t+m-1)x_1^{-t+m-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 2\bar{x}_{m-1} & \dots & (t+m-1)x_{m-1}^{-t+m-2} \end{bmatrix} \quad (2.7)$$

Where t denotes the number of interpolation points x_{n+j}, $j = 0,1,2,...,t-1$, and m denotes the number of collocation points $\bar{x}_j \in [x_n, x_{n+k}]$, the points x_j are chosen the step $\bar{x}_{n+j}$ as well as one or multiple off-grid points.
The following assumptions are made;

1. Although the step size can be variable, for simplicity in our presentation of the analysis in this
Paper, we assume it is constant $h = x_{n+1} - x_n$, $N = \frac{b-a}{h}$ with the steps given by $\left\{x_n / n = 0,1,\dots,\frac{(b-a)}{h}\right\}$, (2.1)
2. That (1.1) has a unique solution and the coefficients $\alpha_j(x)$, $\beta_j(x)$ in (2.1) can be represented by polynomial of the form

$$\alpha_j(x) = \sum_{j=0}^{t+m-1} \alpha_{j,j+1} x^i \qquad j \in \{0,1,\dots,t-1\} \quad (2.2)$$

$$h\beta_j(x) = h\sum_{j=0}^{t+m-1} \beta_{j,j+1} x^i \qquad j \in \{0,1,\dots,m-1\} \quad (2.3)$$

With constant coefficients $\alpha_{j,i+1}$, $h\beta_{j,i+1}$ and collocation conditions:

$$\bar{y}(x_{n+j}) = y_{n+j} \,,\, j \in \{0,1,\dots,t-1\} \quad (2.4)$$

$$\bar{y}'(\bar{x}_j) = f(\bar{x}_j, \bar{y}(\bar{x}_j)) \,,\; j \in \{0,1,\dots,m-1\} \quad (2.5)$$

With these assumptions we obtained an MC polynomial in the form:

$$y(x) = \sum_{j=0}^{t+m-1} \alpha_{j,} x^i, \;\; \alpha_j = \sum_{j=0}^{t-1} c_{i+1,j+1} + \sum_{j=0}^{m-1} c_{i+1,j+1} f_{n+j} \quad (2.6)$$

The parameter required for equation (2.7) to obtain three-step block hybrid method with three off-grid points are k = 3, t = 1 and m = k + 4: $x = \{x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}, x_{n+\frac{5}{2}}, x_{n+3}\}$. Therefore, we get six members schemes. Hence, the hybrid block methods are as follows:

$$y_{n+\frac{1}{2}} = y_n + \frac{1}{120960}h[19087f_n + 65112f_{n+\frac{1}{2}} - 46461f_{n+1} + 37504f_{n+\frac{3}{2}} - 20211f_{n+2} + 6312f_{n+\frac{5}{2}} - 863f_{n+3}]$$

$$y_{n+1} = y_n + \frac{1}{7560}h[1139f_n + 5640f_{n+\frac{1}{2}} - 33f_{n+1} + 1328f_{n+\frac{3}{2}} - 807f_{n+2} + 264f_{n+\frac{5}{2}} - 37f_{n+3}]$$

$$y_{n+\frac{3}{2}} = y_n + \frac{1}{4480}h[685f_n + 3240f_{n+\frac{1}{2}} - 1161f_{n+1} + 2176f_{n+\frac{3}{2}} - 729f_{n+2} + 216f_{n+\frac{5}{2}} - 29f_{n+3}]$$

$$y_{n+2} = y_n + \frac{1}{945}h[143f_n + 696f_{n+\frac{1}{2}} + 192f_{n+1} + 752f_{n+\frac{3}{2}} + 87f_{n+2} + 24f_{n+\frac{5}{2}} - 4f_{n+3}]$$

$$y_{n+\frac{5}{2}} = y_n + \frac{5}{7560}h[743f_n + 3480f_{n+\frac{1}{2}} + 1275f_{n+1} + 3200f_{n+\frac{3}{2}} + 2325f_{n+2} + 1128f_{n+\frac{5}{2}} - 55f_{n+3}]$$

$$y_{n+3} = y_n + \frac{1}{280}h[41f_n + 216f_{n+\frac{1}{2}} + 27f_{n+1} + 272f_{n+\frac{3}{2}} + 27f_{n+2} + 216f_{n+\frac{5}{2}} + 41f_{n+3}] \quad (2.8)$$

The parameter required for equation (2.7) to obtain Four-step block hybrid method with four off-grid points are k = 4, t = 1 and m = k + 5: $x = \{x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}, x_{n+3}, x_{n+\frac{7}{2}}, x_{n+4}\}$ By Using the maple software program and evaluating (2.7) at the grid-points, $x = \{x_{n+\frac{1}{2}}, x = x_{n+1}, x = x_{n+\frac{3}{2}}, x = x_{n+2}, x = x_{n+\frac{5}{2}}, = x_{n+3}, x = x_{n+\frac{7}{2}}, = x_{n+4}\}$ we get eight members schemes. Hence, the hybrid block methods are as follows:

$$y_{n+1} = y_n + \frac{1}{226800}h[32377f_n + 182584f_{n+\frac{1}{2}} - 42494f_{n+1} + 120088f_{n+\frac{3}{2}} - 116120f_{n+2} + 74728f_{n+\frac{5}{2}} - 31154f_{n+3} + 7624f_{n+\frac{7}{2}} - 833f_{n+4}]$$

$$y_{n+\frac{3}{2}} = y_n + \frac{1}{896600}h[12881f_n + 70902f_{n+\frac{1}{2}} + 3438f_{n+1} + 79934f_{n+\frac{3}{2}} - 56160f_{n+2} + 34434f_{n+\frac{5}{2}} - 14062f_{n+3} + 3402f_{n+\frac{7}{2}} - 369f_{n+4}]$$

$$y_{n+2} = y_n + \frac{1}{28350}h[4063f_n + 22576f_{n+\frac{1}{2}} + 244f_{n+1} + 32752f_{n+\frac{3}{2}} - 9080f_{n+2} + 9232f_{n+\frac{5}{2}} - 3956f_{n+3} + 976f_{n+\frac{7}{2}} - 107f_{n+4}]$$

$$y_{n+\frac{5}{2}} = y_n + \frac{5}{290304}h[8341f_n + 46030f_{n+\frac{1}{2}} + 1510f_{n+1} + 63670f_{n+\frac{3}{2}} - 800f_{n+2} + 3186f_{n+\frac{5}{2}} - 9830f_{n+3} + 2290f_{n+\frac{7}{2}} - 245f_{n+4}]$$

$$y_{n+3} = y_n + \frac{1}{28000}h[401f_n + 2232f_{n+\frac{1}{2}} + 18f_{n+1} + 3224f_{n+\frac{3}{2}} - 360f_{n+2} + 2664f_{n+\frac{5}{2}} + 158f_{n+3} + 72f_{n+\frac{7}{2}} - 9f_{n+4}]$$

$$y_{n+\frac{7}{2}} = y_n + \frac{7}{1036800}h[21361f_n + 116662f_{n+\frac{1}{2}} + 6958f_{n+1} + 155134f_{n+\frac{3}{2}} + 7840f_{n+2} + 105154f_{n+\frac{5}{2}} + 74578f_{n+3} + 31882f_{n+\frac{7}{2}} - 1169f_{n+4}]$$

$$y_{n+4} = y_n + \frac{2}{14175}h[989f_n + 5888f_{n+\frac{1}{2}} - 928f_{n+1} + 10496f_{n+\frac{3}{2}} - 4540f_{n+2} + 10496f_{n+\frac{5}{2}} - 928f_{n+3} + 5888f_{n+\frac{7}{2}} + 989f_{n+4}] \quad (2.9)$$

2.2 Stability of Block Method

The equations (2.8) when put together formed the block as:

$$\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_{n+1}\\y_{n+\frac{3}{2}}\\y_{n+2}\\y_{n+\frac{5}{2}}\\y_{n+3}\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+3}\\y_{n+\frac{5}{2}}\\y_{n+2}\\y_{n+\frac{3}{2}}\\y_{n+1}\\y_{n+\frac{1}{2}}\end{bmatrix}+$$

$$h\begin{bmatrix}\frac{2713}{5040}&-\frac{15487}{40320}&\frac{293}{945}&-\frac{6737}{40370}&\frac{263}{5040}&-\frac{863}{120960}\\\frac{47}{63}&\frac{11}{2520}&\frac{166}{945}&-\frac{269}{2520}&\frac{11}{315}&-\frac{37}{7560}\\\frac{81}{112}&\frac{1161}{4480}&\frac{17}{35}&-\frac{729}{4480}&\frac{27}{560}&-\frac{29}{4480}\\\frac{232}{315}&\frac{64}{315}&\frac{752}{945}&\frac{29}{315}&\frac{8}{315}&-\frac{4}{945}\\\frac{725}{1008}&\frac{2125}{8064}&\frac{125}{189}&\frac{3875}{8064}&\frac{235}{1008}&-\frac{275}{24192}\\\frac{27}{35}&\frac{27}{280}&\frac{34}{35}&\frac{27}{280}&\frac{27}{35}&\frac{41}{280}\end{bmatrix}\begin{bmatrix}f_{n+\frac{1}{2}}\\f_{n+1}\\f_{n+\frac{3}{2}}\\f_{n+2}\\f_{n+\frac{5}{2}}\\f_{n+3}\end{bmatrix}+h\begin{bmatrix}0&0&0&0&0&\frac{19087}{120960}\\0&0&0&0&0&\frac{1139}{7560}\\0&0&0&0&0&\frac{137}{896}\\0&0&0&0&0&\frac{19087}{120960}\\0&0&0&0&0&\frac{19087}{120960}\\0&0&0&0&0&\frac{19087}{120960}\end{bmatrix}\begin{bmatrix}f_{n+3}\\f_{n+\frac{5}{2}}\\f_{n+2}\\f_{n+\frac{3}{2}}\\f_{n+1}\\f_{n+\frac{1}{2}}\end{bmatrix} \quad (2.10)$$

The characteristic polynomial of the hybrid block method (2.8) and (2.10) is given as

$\rho(R) = det[RA^0 - A^1]$, Where $A^0 = \begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$

And $A^1 = \begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}$

$$\rho(R) = \det\left[R\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\right]=\det\begin{bmatrix}R&0&0&0&0&-1\\0&R&0&0&0&-1\\0&0&R&0&0&-1\\0&0&0&R&0&-1\\0&0&0&0&R&-1\\0&0&0&0&0&R-1\end{bmatrix}=0$$

$$R\left(R\left(R\left(R\left(R\left(R(R-1)\right)\right)\right)\right)\right)=0,\ \Rightarrow R_1=R_2=R_3=R_4=R_5=0, R_6=1$$

Where $|R_j| \leq 1, j \in \{1, \dots, 6\}$,

The method as a block is zero stable on its own and the hybrid Method is also consistent as it order $p > 1$.

Also, the hybrid block method with four off-grid points, the equation (2.9) when put together formed the block as

$$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_{n+1}\\y_{n+\frac{3}{2}}\\y_{n+2}\\y_{n+\frac{5}{2}}\\y_{n+3}\\y_{n+\frac{7}{2}}\\y_{n+4}\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+4}\\y_{n+\frac{7}{2}}\\y_{n+3}\\y_{n+\frac{5}{2}}\\y_{n+2}\\y_{n+\frac{3}{2}}\\y_{n+1}\\y_{n+\frac{1}{2}}\end{bmatrix}$$

$$+h\begin{bmatrix}\frac{2233547}{2628800}&-\frac{2302297}{3628800}&\frac{2797679}{3628800}&-\frac{31457}{45360}&\frac{1573169}{3628800}&-\frac{645607}{3628800}&\frac{156437}{3628800}&-\frac{33953}{7257600}\\\frac{22823}{28350}&-\frac{21247}{113400}&\frac{15011}{28350}&-\frac{2903}{5670}&\frac{9341}{28350}&-\frac{15577}{113400}&\frac{953}{28350}&-\frac{119}{32400}\\\frac{35451}{44800}&\frac{1719}{44800}&\frac{39967}{44800}&-\frac{351}{560}&\frac{17217}{44800}&-\frac{7031}{44800}&\frac{243}{6400}&-\frac{369}{89600}\\\frac{11288}{14175}&-\frac{122}{14175}&\frac{16376}{14175}&-\frac{908}{2835}&\frac{4616}{14175}&-\frac{11978}{14175}&\frac{488}{14175}&-\frac{107}{28350}\\\frac{115075}{145152}&\frac{3775}{145152}&\frac{159175}{145152}&-\frac{125}{9072}&\frac{85465}{145152}&-\frac{24575}{145152}&\frac{5725}{145152}&-\frac{369}{41472}\\\frac{279}{350}&\frac{9}{1400}&\frac{403}{350}&-\frac{9}{70}&\frac{333}{350}&\frac{79}{1400}&\frac{9}{350}&\frac{9}{2800}\\\frac{408317}{518400}&\frac{24353}{518400}&\frac{542969}{518400}&\frac{343}{6480}&\frac{368039}{518400}&\frac{261023}{518400}&\frac{111587}{518400}&-\frac{8183}{1036800}\\\frac{11776}{14175}&\frac{1856}{14175}&\frac{20992}{14175}&-\frac{1816}{2835}&\frac{20992}{14175}&-\frac{1856}{1400}&\frac{11776}{14175}&\frac{1978}{14175}\end{bmatrix}\begin{bmatrix}f_{n+\frac{1}{2}}\\f_{n+1}\\f_{n+\frac{3}{2}}\\f_{n+2}\\f_{n+\frac{5}{2}}\\f_{n+3}\\f_{n+\frac{7}{2}}\\f_{n+4}\end{bmatrix}$$

$$+h\begin{bmatrix}0&0&0&0&0&0&0&\frac{1070017}{7257600}\\0&0&0&0&0&0&0&\frac{32377}{226800}\\0&0&0&0&0&0&0&\frac{12881}{89600}\\0&0&0&0&0&0&0&\frac{4063}{28350}\\0&0&0&0&0&0&0&\frac{41705}{290304}\\0&0&0&0&0&0&0&\frac{401}{2800}\\0&0&0&0&0&0&0&\frac{149527}{1036800}\\0&0&0&0&0&0&0&\frac{1978}{14175}\end{bmatrix}\begin{bmatrix}f_{n+4}\\f_{n+\frac{7}{2}}\\f_{n+3}\\f_{n+\frac{5}{2}}\\f_{n+2}\\f_{n+\frac{3}{2}}\\f_{n+1}\\f_{n+\frac{1}{2}}\end{bmatrix} \quad (2.11)$$

The characteristic polynomial of the hybrid block methods (2.9) and (2.11) is given as

$\rho(R) = det[RA^0 - A^1]$, Where

$$A^0=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix} \quad and \quad A^1=\begin{bmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{bmatrix}$$

$$\text{Therefore,}\det\left(RA^0 - A^1\right) = det\begin{bmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & R & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & R & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{bmatrix}$$

$$= R\left(R\left(R\left(R\left(R\left(R\left(R(R-1)\right)\right)\right)\right)\right)\right) = 0 \quad \Rightarrow R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 0, R_8 = 1$$

$Where\ |R_j| \leq 1, j \in \{1, \ldots, 6\}$, The method as a block is zero stable on its own and the hybrid method is also consistent as it order $p > 1$.

3. CONVERGENCE ANALYSIS

3.1 Order and Error Constants of the Hybrid Block Methods

The hybrid block methods which are obtained in a block form with the help of maple software have the following order and error constants for each block hybrid method.

The method $k = 3$ with three off- grid points is of order 7 and has error constants

$$C_8 = \left[\frac{275}{6193152}, \frac{1}{30240}, \frac{9}{229376}, \frac{1}{30240}, \frac{275}{6193152}, \frac{-9}{716800}\right]^T$$

The method $k = 4$ with four off- grid points is of order 9 and has error constants

$$C_{10} = \left[\frac{8183}{1061683200}, \frac{9}{1433600}, \frac{25}{3670016}, \frac{47}{7257600}, \frac{25}{3670016}, \frac{9}{1433600}, \frac{8183}{1061683200}, \frac{-37}{14968800}\right]^T$$

3.2 Region of Absolute Stability

Using the MATLAB package, we were able to plot the stability region of the block method (see the Figs. 1 and 2). This is done by reformulating the block method as general linear method to obtain the values of the matrices according to [6, 7, 8, 9, and 10]. These matrices are substituted into the stability matrix and using MATLAB software, the absolute stability regions of the new methods are plotted as shown in the Figs. 1 and 2.

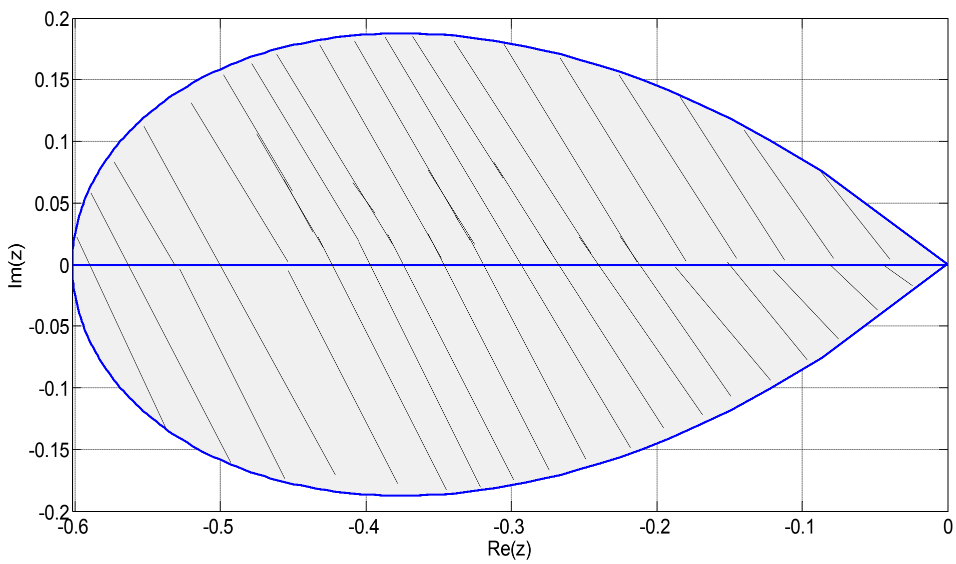

Fig. 1. Stability region of the block hybrid method for $k = 3$ with three off-grid points

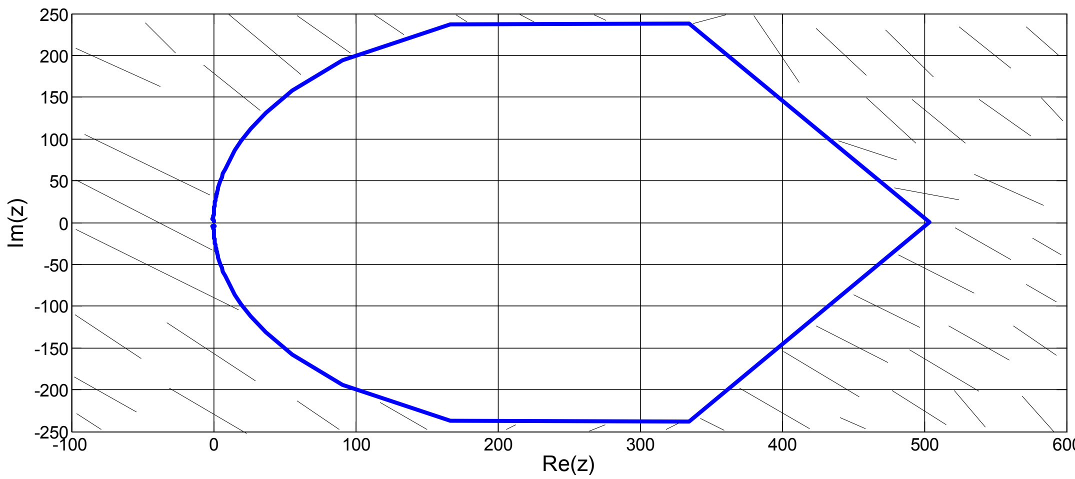

Fig. 2. Stability region of the block hybrid method for $k = 4$ with four off-grid points

Table 1. Absolute error for experiment 1

Y	$A_{(\alpha)}$ stable hybrid block three off grid points	A-stable hybrid block with four off-grid points
0.1	6.67×10^{-2}	3.55×10^{-2}
0.2	6.67×10^{-2}	1.41×10^{-2}
0.3	9.99×10^{-2}	3.52×10^{-2}
0.4	6.67×10^{-2}	9.79×10^{-1}
0.5	6.67×10^{-2}	3.47×10^{-2}
0.6	9.99×10^{-1}	1.38×10^{-2}
0.7	6.67×10^{-2}	3.44×10^{-2}
0.8	6.67×10^{-2}	9.57×10^{-1}
0.9	9.99×10^{-1}	3.40×10^{-2}
1.0	6.67×10^{-2}	1.35×10^{-1}
1.1	6.67×10^{-2}	3.37×10^{-2}
1.2	9.99×10^{-1}	9.37×10^{-1}

Table 2. Absolute error for experiment 2

Y	$A_{(\alpha)}$ stable hybrid block with three off- grid points		A-stable hybrid block with four off-grid points	
	y_1	y_2	y_1	y_2
0.1	7.50×10^{-1}	8.40×10^{-1}	7.50×10^{-1}	5.31×10^{-2}
0.2	5.36×10^{-1}	8.09×10^{-2}	5.36×10^{-1}	2.86×10^{-2}
0.3	3.69×10^{-1}	1.01×10^{-1}	3.69×10^{-1}	4.74×10^{-2}
0.4	2.38×10^{-1}	7.62×10^{-2}	2.86×10^{-2}	1.01×10^{-1}
0.5	1.37×10^{-1}	7.39×10^{-2}	1.37×10^{-1}	4.29×10^{-2}
0.6	6.00×10^{-2}	9.95×10^{-1}	6.00×10^{-2}	1.43×10^{-2}
0.7	1.86×10^{-3}	7.19×10^{-2}	1.86×10^{-3}	4.10×10^{-2}
0.8	4.12×10^{-2}	7.10×10^{-2}	4.12×10^{-2}	1.00×10^{1}
0.9	7.26×10^{-2}	1.00×10^{1}	7.22×10^{-2}	3.92×10^{-2}
1.0	9.45×10^{-2}	6.95×10^{-2}	9.45×10^{-2}	1.72×10^{-2}
1.1	1.15×10^{-1}	6.90×10^{-2}	1.09×10^{-1}	3.81×10^{-2}
1.2	1.18×10^{-1}	1.00×10^{1}	1.18×10^{-1}	1.00×10^{1}

3.3 Numerical Implementation

To study the efficiency of the block method for *k=3*, we present some numerical example widely used by some authors such as [4,5,10].

Experiment 1: $y' = \lambda y$, *Where* $h = 0.1$, $\lambda = -1000C$, $x \in [0, 1.2]$ ***Exact Solution:*** $y(x) = e^{-10000x}$

Experiment 2: $y'_{1=-y_1+95y_2}$, $y'_{2=-y_1-97y_2}$, Where $h = \frac{1}{10}$, $y_1(0) = 1$, $y_2(0) = 1$

$\boldsymbol{Exact\ solution}$:

$y_{1(x)=}\frac{95}{47}e^{-2x}-\frac{48}{47}e^{-96x}$, $y_2(x)=-\frac{48}{47}e^{-96x}-\frac{1}{47}e^{-2x}$, $with\ stiff\ ratio\ 4.8 \times 10^1$

4. CONCLUSION

It is obvious from the results presented in the tables above that our methods are indeed accurate, and yield a good results, since they handle stiff equations. Also in terms of stability analysis, the method with three off-grid points is $A_{(\alpha)}$ -stable and the method with four off-grid points is A-stable.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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